EL POTENCIAL ELECTROSTÁTICO.

5.2 (c) Un dipolo eléctrico, generalización a multipolos

Potential Due to a Dipole

Two equal charges of opposite sign, $\pm q$, separated by a distance d, constitute an electric dipole; see Section 28-3. The electric dipole moment **p** has the magnitude qd and points from the negative charge to the positive charge. Here we derive an expression for the electric potential V due to a dipole.

A point P is specified by giving the quantities r and θ in Fig. 10. From symmetry, it is clear that the potential does not change as point P rotates about the z axis, r and θ being fixed. (Equivalently, consider what would happen if the dipole were rotated about the z axis: there would be no change in the physical situation.) Thus if we find V for points in the plane of Fig. 10, we have found V for all points in space. Applying Eq. 19 gives

$$V_P = \sum_i V_i = V_1 + V_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_1} + \frac{-q}{r_2}\right) = \frac{q}{4\pi\epsilon_0}\,\frac{r_2 - r_1}{r_1 r_2}, \qquad (20)$$

which is an exact relationship.

For naturally occurring dipoles, such as many molecules, the observation point P is located very far from the dipole, such that $r \gg d$. Under this condition, we can deduce from Fig. 10 that

$$r_2 - r_1 \approx d\cos\theta \quad \text{and} \quad r_1 r_2 \approx r^2,$$

and the potential reduces to

$$V \approx \frac{q}{4\pi\epsilon_0}\,\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\,\frac{p\cos\theta}{r^2}. \qquad (21)$$

Note that $V = 0$ everywhere in the equatorial plane ($\theta = 90°$). This means that the electric field of the dipole does no work when a test charge moves from infinity along a line in the midplane of the dipole (for instance, the

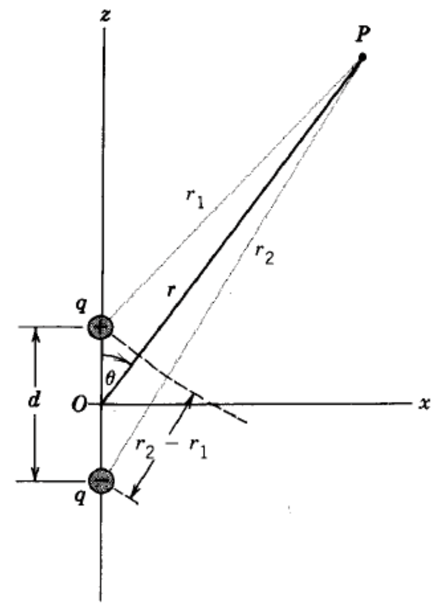

Figure 10 A point P in the field of an electric dipole.

x axis in Fig. 10). For a given r, the potential has its greatest positive value for $\theta = 0°$ and its greatest negative value for $\theta = 180°$. Note that V does not depend separately on q and d but only on their product p.

Although certain molecules, such as water, do have permanent electric dipole moments (see Fig. 18 of Chapter 28), individual atoms and many other molecules do not. However, dipole moments may be induced by placing *any* atom or molecule in an external electric field. The action of the field, as Fig. 11 shows, is to separate the centers of positive and negative charge. We say that the atom becomes *polarized* and acquires an *induced electric dipole moment*. Induced dipole moments disappear when the electric field is removed.

Electric dipoles are important in situations other than atomic and molecular ones. Radio and TV antennas are often in the form of a metal wire or rod in which electrons surge back and forth periodically. At a certain time one end of the wire or rod is negative and the other end positive. Half a cycle later the polarity of the ends is exactly reversed. This is an *oscillating* electric dipole. It is so named because its dipole moment changes in a periodic way with time.

Sample Problem 8 An *electric quadrupole* consists of two electric dipoles so arranged that they almost, but not quite, cancel each other in their electric effects at distant points (see Fig. 12). Calculate $V(r)$ for points on the axis of this quadrupole.

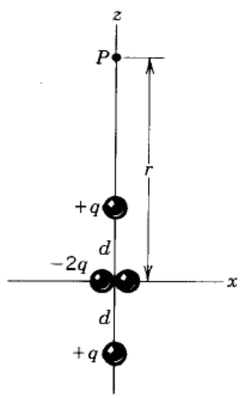

Figure 12 Sample Problem 8. An electric quadrupole, consisting of two oppositely directed electric dipoles.

Solution Applying Eq. 19 to Fig. 12 yields

$$V=\sum_i V_i=\frac{1}{4\pi\epsilon_0}\left(\frac{q}{r-d}+\frac{-2q}{r}+\frac{q}{r+d}\right)$$
$$=\frac{1}{4\pi\epsilon_0}\frac{2qd^2}{r(r^2-d^2)}=\frac{1}{4\pi\epsilon_0}\frac{2dq^2}{r^3(1-d^2/r^2)}\,.$$

Because $d \ll r$, we can neglect d^2/r^2 compared with 1, in which case the potential becomes

$$V=\frac{1}{4\pi\epsilon_0}\frac{Q}{r^3}, \tag{22}$$

where Q $(=2qd^2)$ is the *electric quadrupole moment* of the charge assembly of Fig. 12. Note that V varies (1) as $1/r$ for a point charge (see Eq. 18), (2) as $1/r^2$ for a dipole (see Eq. 21), and (3) as $1/r^3$ for a quadrupole (see Eq. 22).

Note too that (1) a dipole is two equal and opposite charges that do not quite coincide in space so that their electric effects at distant points do not quite cancel, and (2) a quadrupole is two equal and opposite dipoles that do not quite coincide in space so that their electric effects at distant points again do not quite cancel. We can continue to construct more complex assemblies of electric charges. This process turns out to be useful, because the electric potential of *any* charge distribution can be represented as a series of terms in increasing powers of $1/r$. The $1/r$ part, called the *monopole* term, depends on the net charge of the distribution, and the succeeding terms ($1/r^2$, the *dipole* term; $1/r^3$, the *quadrupole* term; and so on) indicate how the charge is distributed. This type of analysis is called an *expansion in multipoles.*