CHAPTER 32

CURRENT AND RESISTANCE

The previous five chapters dealt with electrostatics, *that is, with charges at rest. With this chapter we begin our study of* electric currents, *that is, of charges in motion.*

Examples of electric currents abound, ranging from the large currents that constitute lightning strokes to the tiny nerve currents that regulate our muscular activity. We are familiar with currents resulting from charges flowing through solid conductors (household wiring, light bulbs), semiconductors (integrated circuits), gases (fluorescent lamps), liquids (automobile batteries), and even evacuated spaces (TV picture tubes).

On a global scale, charged particles trapped in the Van Allen radiation belts surge back and forth above the atmosphere between the north and the south magnetic poles. On the scale of the solar system, enormous currents of protons, electrons, and ions travel radially outward from the Sun as the solar wind. On the galactic scale, cosmic rays, which are largely energetic protons, stream through the galaxy.

32-1 ELECTRIC CURRENT

The free electrons in an isolated metallic conductor, such as the length of wire illustrated in Fig. 1*a*, are in random motion like the molecules of a gas confined to a container. They have no net directed motion along the wire. If we pass a hypothetical plane through the wire, the rate at which electrons cross that plane in one direction is equal to the rate at which they cross in the other direction; the *net* rate is zero. (Here we assume our observation time is long enough so that the small statistical fluctuations in the number of electrons crossing the plane average to zero. In some cases, the fluctuations can be important. For example, they contribute to the electrical noise in circuits.)

Whether the conductor of Fig. 1*a* is charged or uncharged, there is no net flow of charge in its interior. In the absence of an externally applied field, no electric field exists within the volume of the conductor or parallel to its surface. Even though an abundance of conduction electrons is available, there is no force on the electrons and no net flow of charge.

In Fig. 1*b*, a battery has been connected across the ends of the conductor. If the battery maintains a potential difference V and the wire has length L, then an electric field of magnitude V/L is established in the conductor. This electric field **E** acts on the electrons and gives them a net motion in the direction opposite to **E**. If the battery could maintain the potential difference, then the charges would continue to circulate indefinitely. In reality, a battery can maintain the current only as long as it is able to convert chemical energy to electrical energy; eventually the battery's source of energy is exhausted, and the potential difference cannot be maintained.

The existence of an electric field inside a conductor does not contradict Section 29-4, in which we asserted that **E** equals zero inside a conductor. In that section, which dealt with a state in which all net motion of charge had stopped (electrostatics), we assumed that the conductor was insulated and that no potential difference was deliberately maintained between any two points on it, as by a battery. In this chapter, which deals with charges in motion, we relax this restriction.

If a *net* charge dq passes through any surface in a time interval dt, we say that an *electric current* i has been established, where

$$i = dq/dt. \qquad (1)$$

For current in a wire, we take dq to be the charge that passes through a cross section in the time dt.

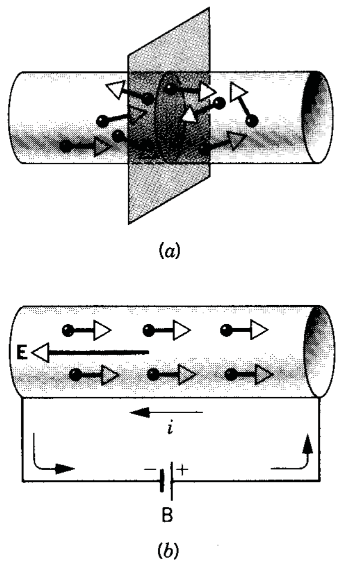

Figure 1 (*a*) In an isolated conductor, the electrons are in random motion. The net flow of charge across an arbitrary plane is zero. (*b*) A battery **B** connected across the conductor sets up an electric field **E**, and the electrons acquire a net motion due to the field.

Note that we require a *net* charge dq to flow for a current to be established. In Fig. 1*a*, equal numbers of electrons are flowing in both directions across the plane; even though there may be a considerable number of electrons flowing across the plane, the current is zero. For another example, the flow of water through a garden hose does not give rise to an electric current according to our definition because the electrically neutral molecules flowing across any surface carry equal positive and negative charges; thus the net flow of charge is zero.

The SI unit of current is the *ampere* (abbreviation A). According to Eq. 1, we have

$$1 \text{ ampere} = 1 \text{ coulomb/second}.$$

You will recall from Section 27-4 that Eq. 1 provides the definition of the coulomb, because the ampere is a SI base unit (see Appendix A). The determination of this fundamental quantity is discussed in Section 35-4.

The net charge that passes through the surface in any time interval is found by integrating the current:

$$q = \int i\, dt. \tag{2}$$

If the current is constant in time, then the charge q that flows in time t determines the current i according to

$$i = q/t. \tag{3}$$

In this chapter we consider only currents that are constant in time; currents that vary with time are considered in Chapter 33. Although there are many different kinds of currents (some of which are mentioned in the introduction), in this chapter we restrict our discussion to electrons moving through solid conductors.

We assume that, under steady conditions, charge does not collect at or drain away from any point in our idealized wire. In the language of Section 18-2, there are no sources or sinks of charge in the wire. When we made this assumption in our study of incompressible fluids, we concluded that the rate at which the fluid flows past any cross section of a pipe is the same even if the cross section varies. The fluid flows faster where the pipe is smaller and slower where it is larger, but the volume rate of flow, measured perhaps in liters/second, remains constant. In the same way, *the electric current i is the same for all cross sections of a conductor, even though the cross-sectional area may be different at different points.*

Although in metals the charge carriers are electrons, in electrolytes or in gaseous conductors (plasmas) they may also be positive or negative ions, or both. We need a convention for labeling the direction of current because charges of opposite sign move in opposite directions in a given field. A positive charge moving in one direction is equivalent in nearly all external effects to a negative charge moving in the opposite direction. Hence, for simplicity and algebraic consistency, we adopt the following convention:

The direction of current is the direction that positive charges would move, even if the actual charge carriers are negative.

If the charge carriers are negative, they simply move opposite to the direction of the current arrow (see Fig. 1*b*).

Under most circumstances, we analyze electric circuits based on an assumed direction for the current, without taking into account whether the actual charge carriers are positive or negative. In rare cases (see, for example, the Hall effect in Section 34-4) we must take into account the sign of the charge carriers.

Even though we assign it a direction, current is a scalar and not a vector. The arrow that we draw to indicate the direction of the current merely shows the sense of the charge flow through the wire and is *not* to be taken as a vector. Current does not obey the laws of vector addition, as can be seen from Fig. 2. The current i_1 in wire 1 divides

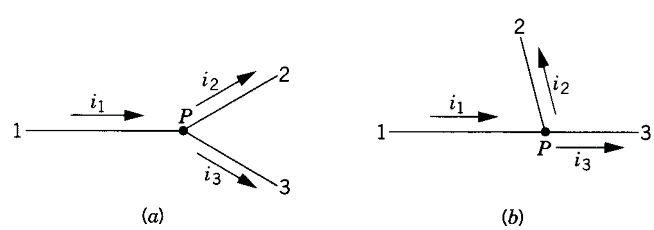

Figure 2 (*a*) At point *P*, the current i_1 divides into currents i_2 and i_3, such that $i_1 = i_2 + i_3$. (*b*) Changing the direction of the wires does not change the way the currents add, illustrating that currents add like scalars, not like vectors.

into two branches i_2 and i_3 in wires 2 and 3, such that $i_1 = i_2 + i_3$. Changing the directions of the wires does not change the way the currents are added, as it would if they added like vectors.

32-2 CURRENT DENSITY

The current i is a characteristic of a particular conductor. It is a macroscopic quantity, like the mass of an object, the volume of an object, or the length of a rod. A related microscopic quantity is the *current density* $\mathbf{j}$. It is a vector and is characteristic of a point inside a conductor rather than of the conductor as a whole. If the current is distributed uniformly across a conductor of cross-sectional area A, as in Fig. 3, the magnitude of the current density for all points on that cross section is

$$j = i/A. \tag{4}$$

The vector $\mathbf{j}$ at any point is oriented in the direction that a positive charge carrier would move at that point. An electron at that point moves in the direction $-\mathbf{j}$. In Fig. 3, $\mathbf{j}$ is a constant vector and points to the left; the electrons drift to the right.

In general, for a particular surface (which need not be plane) that cuts across a conductor, i is the flux of the vector $\mathbf{j}$ over that surface, or

$$i = \int \mathbf{j} \cdot d\mathbf{A}, \tag{5}$$

where $d\mathbf{A}$ is an element of surface area and the integral is done over the surface in question. The vector $d\mathbf{A}$ is taken to be perpendicular to the surface element such that $\mathbf{j} \cdot d\mathbf{A}$ is positive, giving a positive current i. Equation 4 (written as $i = jA$) is a special case of Eq. 5 in which the surface of integration is a plane cross section of the conductor and in which $\mathbf{j}$ is constant over this surface and at right angles to it. However, we may apply Eq. 5 to *any* surface through which we wish to know the current. Equation 5 shows clearly that i is a scalar because the integrand $\mathbf{j} \cdot d\mathbf{A}$ is a scalar.

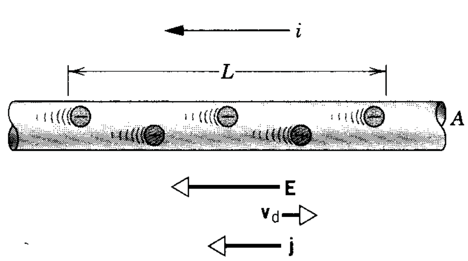

Figure 3 The electric field causes electrons to drift to the right. The conventional current (the hypothetical direction of flow of positive charge) is to the left. The current density $\mathbf{j}$ is likewise drawn as if the charge carriers were positive, so that $\mathbf{j}$ and $\mathbf{E}$ are in the same direction.

The electric field exerts a force ($= -e\mathbf{E}$) on the electrons in a conductor but this force does not produce a *net* acceleration because the electrons keep colliding with the atoms or ions that make up the conductor. This array of ions, coupled together by strong springlike forces of electromagnetic origin, is called the *lattice* (see Fig. 11 of Chapter 14). The overall effect of the collisions is to transfer kinetic energy from the accelerating electrons into vibrational energy of the lattice. The electrons acquire a constant average *drift speed* v_d in the direction $-\mathbf{E}$. There is a close analogy to a ball falling in a uniform gravitational field $\mathbf{g}$ at a constant terminal speed through a viscous fluid. The gravitational force ($m\mathbf{g}$) acting on the falling ball does not increase the ball's kinetic energy (which is constant); instead, energy is transferred to the fluid by molecular collisions and produces a small rise in temperature.

We can compute the drift speed v_d of charge carriers in a conductor from the current density j. Figure 3 shows the conduction electrons in a wire moving to the right at an assumed constant drift speed v_d. The number of conduction electrons in a length L of the wire is nAL, where n is the number of conduction electrons per unit volume and AL is the volume of the length L of the wire. A charge of magnitude

$$q = (nAL)e$$

passes out of this segment of the wire, through its right end, in a time t given by

$$t = \frac{L}{v_d}.$$

The current i is

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d.$$

Solving for v_d and recalling that $j = i/A$ (Eq. 4), we obtain

$$v_d = \frac{i}{nAe} = \frac{j}{ne}. \tag{6}$$

Since both v_d and j are vectors, we can rewrite Eq. 6 as a vector equation. We follow our adopted convention for positive current density, which means we must take the direction of $\mathbf{j}$ to be opposite to that of $\mathbf{v}_d$. The vector equivalent of Eq. 6 is therefore

$$\mathbf{j} = -ne\mathbf{v}_d. \tag{7}$$

Figure 3 shows that, for electrons, these vectors are indeed in opposite directions.

As the following sample problems illustrate, the drift speed in typical conductors is quite small, often of the order of cm/s. In contrast, the random thermal motion of conduction electrons in a metal takes place with typical speeds of 10^6 m/s.

Sample Problem 1 One end of an aluminum wire whose diameter is 2.5 mm is welded to one end of a copper wire whose diameter is 1.8 mm. The composite wire carries a steady current i of 1.3 A. What is the current density in each wire?

Solution We may take the current density as (a different) constant within each wire except for points near the junction. The current density is given by Eq. 4,

$$j = \frac{i}{A}.$$

The cross-sectional area A of the aluminum wire is

$$A_{\text{Al}} = \frac{1}{4}\pi d^2 = (\pi/4)(2.5 \times 10^{-3}\ \text{m})^2 = 4.91 \times 10^{-6}\ \text{m}^2$$

so that

$$j_{\text{Al}} = \frac{1.3\ \text{A}}{4.91 \times 10^{-6}\ \text{m}^2} = 2.6 \times 10^5\ \text{A/m}^2 = 26\ \text{A/cm}^2.$$

As you can verify, the cross-sectional area of the copper wire is $2.54 \times 10^{-6}\ \text{m}^2$, so that

$$j_{\text{Cu}} = \frac{1.3\ \text{A}}{2.54 \times 10^{-6}\ \text{m}^2} = 5.1 \times 10^5\ \text{A/m}^2 = 51\ \text{A/cm}^2.$$

The fact that the wires are of different materials does not enter here.

Sample Problem 2 What is the drift speed of the conduction electrons in the copper wire of Sample Problem 1?

Solution The drift speed is given by Eq. 6,

$$v_{\text{d}} = \frac{j}{ne}.$$

In copper, there is very nearly one conduction electron per atom on the average. The number n of electrons per unit volume is therefore the same as the number of atoms per unit volume and is given by

$$\frac{n}{N_{\text{A}}} = \frac{\rho_{\text{m}}}{M} \quad \text{or} \quad \frac{\text{atoms/m}^3}{\text{atoms/mol}} = \frac{\text{mass/m}^3}{\text{mass/mol}}.$$

Here ρ_{m} is the (mass) density of copper, N_{A} is the Avogadro constant, and M is the molar mass of copper.* Thus

$$n = \frac{N_{\text{A}}\rho_{\text{m}}}{M} = \frac{(6.02 \times 10^{23}\ \text{electrons/mol})(8.96 \times 10^3\ \text{kg/m}^3)}{63.5 \times 10^{-3}\ \text{kg/mol}}$$
$$= 8.49 \times 10^{28}\ \text{electrons/m}^3.$$

We then have

$$v_{\text{d}} = \frac{5.1 \times 10^5\ \text{A/m}^2}{(8.49 \times 10^{28}\ \text{electrons/m}^3)(1.60 \times 10^{-19}\ \text{C/electron})}$$
$$= 3.8 \times 10^{-5}\ \text{m/s} = 14\ \text{cm/h}.$$

You should be able to show that for the aluminum wire, $v_{\text{d}} = 2.7 \times 10^{-5}\ \text{m/s} = 9.7\ \text{cm/h}$. Can you explain, in physical terms, why the drift speed is smaller in aluminum than in copper, even though the two wires carry the same current?

* We use the subscript m to make it clear that the density referred to here is a mass density (kg/m^3), not a charge density (C/m^3).

If the electrons drift at such a low speed, why do electrical effects seem to occur immediately after a switch is thrown, such as when you turn on the room lights? Confusion on this point results from not distinguishing between the drift speed of the electrons and the speed at which *changes* in the electric field configuration travel along wires. This latter speed approaches that of light. Similarly, when you turn the valve on your garden hose, with the hose full of water, a pressure wave travels along the hose at the speed of sound in water. The speed at which the water moves through the hose—measured perhaps with a dye marker—is much lower.

Sample Problem 3 A strip of silicon, of width $w = 3.2$ mm and thickness $d = 250\ \mu\text{m}$, carries a current i of 190 mA. The silicon is an *n-type semiconductor*, having been "doped" with a controlled amount of phosphorus impurity. The doping has the effect of greatly increasing n, the number of charge carriers (electrons, in this case) per unit volume, as compared with the value for pure silicon. In this case, $n = 8.0 \times 10^{21}\ \text{m}^{-3}$. (*a*) What is the current density in the strip? (*b*) What is the drift speed?

Solution (*a*) From Eq. 4,

$$j = \frac{i}{wd} = \frac{190 \times 10^{-3}\ \text{A}}{(3.2 \times 10^{-3}\ \text{m})(250 \times 10^{-6}\ \text{m})}$$
$$= 2.4 \times 10^5\ \text{A/m}^2.$$

(*b*) From Eq. 6,

$$v_{\text{d}} = \frac{j}{ne} = \frac{2.4 \times 10^5\ \text{A/m}^2}{(8.0 \times 10^{21}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})}$$
$$= 190\ \text{m/s}.$$

The drift speed (190 m/s) of the electrons in this silicon semiconductor is much greater than the drift speed (3.8×10^{-5} m/s) of the conduction electrons in the metallic copper conductor of Sample Problem 2, even though the current densities are similar. The number of charge carriers in this semiconductor ($8.0 \times 10^{21}\ \text{m}^{-3}$) is much smaller than the number of charge carriers in the copper conductor ($8.49 \times 10^{28}\ \text{m}^{-3}$). The smaller number of charge carriers must drift faster in the semiconductor if they are to establish the same current density that the greater number of charge carriers establish in copper.

32-3 RESISTANCE, RESISTIVITY, AND CONDUCTIVITY

If we apply the same potential difference between the ends of geometrically similar rods of copper and of wood, very different currents result. The characteristic of the conductor that enters here is its *resistance*. We determine the resistance of a conductor between two points by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = V/i. \qquad (8)$$

If V is in volts and i in amperes, the resistance R is in volts/ampere, which is given the name of *ohms* (abbreviation Ω), such that

$$1 \text{ ohm} = 1 \text{ volt/ampere}.$$

A conductor whose function in a circuit is to provide a specified resistance is called a *resistor* (symbol -WW-).

The flow of charge through a conductor is often compared with the flow of water through a pipe as a result of a difference in pressure between the ends of the pipe, established perhaps by a pump. The pressure difference is analogous to the potential difference between the ends of a conductor, established perhaps by a battery. The rate of flow of water (liters/second, say) is analogous to the rate of flow of charge (coulombs/second, or amperes). The rate of flow of water for a given pressure difference is determined by the nature of the pipe: its length, cross section, and solid interior impediments (for instance, gravel in the pipe). These characteristics of the pipe are analogous to the resistance of a conductor.

The ohm is not a SI base unit (see Appendix A); no primary standard of the ohm is kept and maintained. However, resistance is such an important quantity in science and technology that a *practical reference standard* is maintained at the National Institute of Standards and Technology. Since January 1, 1990, this *representation of the ohm* (as it is known) has been based on the *quantum Hall effect* (see Section 34-4), a precise and highly reproducible quantum phenomenon that is independent of the properties of any particular material.

Related to resistance is the *resistivity* ρ, which is a characteristic of a material rather than of a particular specimen of a material; it is defined by

$$\rho = \frac{E}{j}. \tag{9}$$

The units of ρ are those of E (V/m) divided by j (A/m^2), which are equivalent to Ω·m. Figure 3 indicates that E and j are vectors, and we can write Eq. 9 in vector form as

$$\mathbf{E} = \rho \mathbf{j}. \tag{10}$$

Equations 9 and 10 are valid only for *isotropic* materials, whose electrical properties are the same in all directions.

The resistivity of copper is 1.7×10^{-8} Ω·m; that of fused quartz is about 10^{16} Ω·m. Few physical properties are measurable over such a range of values. Table 1 lists resistivities for some common materials.

Some substances cannot readily be classified as conductors or insulators. Plastics generally have large resistivities that would lead us to classify them with the insulators. For example, household electrical wiring normally uses plastic for insulation. However, by doping plastics with certain chemicals, their conductivity can match that of copper.*

* See "Plastics that Conduct Electricity," by Richard B. Kaner and Alan G. MacDiarmid, *Scientific American*, February 1988, p. 106.

TABLE 1 RESISTIVITY OF SOME MATERIALS AT ROOM TEMPERATURE (20°C)

Material	*Resistivity,* ρ (Ω·m)	*Temperature Coefficient of Resistivity* $\bar{\alpha}$ (per C°)
Typical Metals		
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
Manganin[a]	48.2×10^{-8}	0.002×10^{-3}
Typical Semiconductors		
Silicon pure	2.5×10^{3}	-70×10^{-3}
Silicon *n*-type[b]	8.7×10^{-4}	
Silicon *p*-type[c]	2.8×10^{-3}	
Typical Insulators		
Glass	$10^{10}-10^{14}$	
Polystyrene	$> 10^{14}$	
Fused quartz	$\approx 10^{16}$	

[a] An alloy specifically designed to have a small value of α.
[b] Pure silicon "doped" with phosphorus impurities to a charge carrier density of 10^{23} m^{-3}.
[c] Pure silicon "doped" with aluminum impurities to a charge carrier density of 10^{23} m^{-3}.

Sometimes we prefer to speak of the *conductivity* σ of a material rather than its resistivity. These are reciprocal quantities, related by

$$\sigma = 1/\rho. \tag{11}$$

The SI units of σ are (Ω·m)$^{-1}$. Equation 10 can be written in terms of the conductivity as

$$\mathbf{j} = \sigma \mathbf{E}. \tag{12}$$

If we know the resistivity ρ of a material, we should be able to calculate the resistance R of a particular piece of the material. Consider a cylindrical conductor, of cross-sectional area A and length L carrying a steady current i with a potential difference V between its ends (see Fig. 4). If the cylinder cross sections at each end are equipotential surfaces, the electric field and the current density are constant for all points in the cylinder and have the values

$$E = \frac{V}{L} \quad \text{and} \quad j = \frac{i}{A}.$$

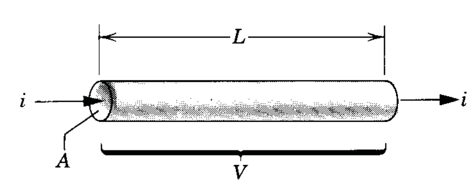

Figure 4 A potential difference V is applied across a cylindrical conductor of length L and cross-sectional area A, establishing a current i.

The resistivity ρ is

$$\rho = \frac{E}{j} = \frac{V/L}{i/A}.$$

But V/i is the resistance R, which leads to

$$R = \rho\frac{L}{A}. \tag{13}$$

We stress that Eq. 13 applies only to a homogeneous, isotropic conductor of uniform cross section subject to a uniform electric field.

Sample Problem 4 A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. (*a*) What is the resistance of the block measured between the two square ends? (*b*) What is the resistance between two opposing rectangular faces? The resistivity of iron at room temperature is $9.68 \times 10^{-8}\ \Omega\cdot\text{m}$.

Solution (*a*) The area of a square end is $(1.2 \times 10^{-2}\ \text{m})^2$ or $1.44 \times 10^{-4}\ \text{m}^2$. From Eq. 13,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(0.15\ \text{m})}{1.44 \times 10^{-4}\ \text{m}^2} = 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega.$$

(*b*) The area of a rectangular face is $(1.2 \times 10^{-2}\ \text{m})(0.15\ \text{m})$ or $1.80 \times 10^{-3}\ \text{m}^2$. From Eq. 13,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(1.2 \times 10^{-2}\ \text{m})}{1.80 \times 10^{-3}\ \text{m}^2} = 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega.$$

We assume in each case that the potential difference is applied to the block in such a way that the surfaces between which the resistance is desired are equipotentials. Otherwise, Eq. 13 would not be valid.

Microscopic and Macroscopic Quantities *(Optional)*

V, i, and R are *macroscopic* quantities, applying to a particular body or extended region. The corresponding *microscopic* quantities are **E**, **j**, and ρ (or σ); they have values at every point in a body. The macroscopic quantities are related by Eq. 8 ($V = iR$) and the microscopic quantities by Eqs. 9, 10, and 12.

The macroscopic quantities can be found by integrating over the microscopic quantities, using relations already given, namely,

$$i = \int \mathbf{j}\cdot d\mathbf{A}$$

and

$$V_{ab} = -V_{ba} = \int_a^b \mathbf{E}\cdot d\mathbf{s}.$$

The current integral is a surface integral, carried out over any cross section of the conductor. The field integral is a line integral carried out along an arbitrary line drawn along the conductor, connecting any two equipotential surfaces, identified by a and b. For a long wire connected to a battery, equipotential surface a might be chosen as a cross section of the wire near the positive battery terminal, and b might be a cross section near the negative terminal.

We can express the resistance of a conductor between a and b in microscopic terms by dividing the two equations:

$$R = \frac{V_{ab}}{i} = \frac{\int_a^b \mathbf{E}\cdot d\mathbf{s}}{\int \mathbf{j}\cdot d\mathbf{A}}.$$

If the conductor is a long cylinder of cross section A and length L, and if points a and b are its ends, the above equation for R reduces to

$$R = \frac{EL}{jA} = \rho\frac{L}{A},$$

which is Eq. 13.

The macroscopic quantities V, i, and R are of primary interest when we are making electrical measurements on real conducting objects. They are the quantities whose values are indicated on meters. The microscopic quantities **E**, **j**, and ρ are of primary importance when we are concerned with the fundamental behavior of matter (rather than of specimens of matter), as we usually are in the research area of *solid state* (or *condensed matter*) physics. Section 32-5 accordingly deals with an atomic view of the *resistivity* of a metal and not of the *resistance* of a metallic specimen. The microscopic quantities are also important when we are interested in the interior behavior of irregularly shaped conducting objects. ■

Temperature Variation of Resistivity *(Optional)*

Figure 5 shows a summary of some experimental measurements of the resistivity of copper at different temperatures. For practical use of this information, it would be helpful to express it in the form of an equation. Over a limited range of temperature, the relationship between resistivity and temperature is nearly linear. We can fit a straight line to any selected region of Fig. 5, using two points to determine the slope of the line. Choosing a reference point, such as that labeled T_0, ρ_0 in the figure, we can

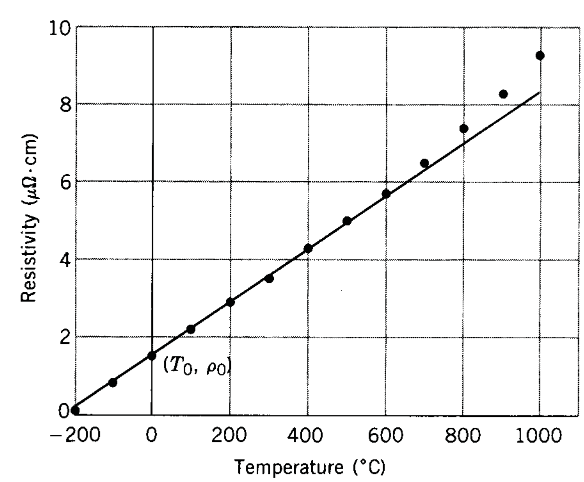

Figure 5 The dots show selected measurements of the resistivity of copper at different temperatures. Over any given range of temperature, the variation in the resistivity with T can be approximated by a straight line; for example, the line shown fits the data from about −100°C to 400°C.

express the resistivity ρ at an arbitrary temperature T from the empirical equation of the straight line in Fig. 5, which is

$$\rho - \rho_0 = \rho_0 \bar{\alpha}(T - T_0). \tag{14}$$

[This expression is very similar to that for linear thermal expansion ($\Delta L = \alpha L \, \Delta T$), which we introduced in Section 22-5.] We have written the slope of this line as $\rho_0 \bar{\alpha}$. If we solve Eq. 14 for $\bar{\alpha}$, we obtain

$$\bar{\alpha} = \frac{1}{\rho_0} \frac{\rho - \rho_0}{T - T_0}. \tag{15}$$

The quantity $\bar{\alpha}$ is the *mean (or average) temperature coefficient of resistivity* over the region of temperature between the two points used to determine the slope of the line. We can define a more general temperature coefficient of resistivity as

$$\alpha = \frac{1}{\rho} \frac{d\rho}{dT}, \tag{16}$$

which is the fractional change in resistivity $d\rho/\rho$ per change in temperature dT. That is, α gives the dependence of resistivity on temperature *at a particular temperature*, while $\bar{\alpha}$ gives the average dependence *over a particular interval*. The coefficient α is in general dependent on temperature.

For most practical purposes, Eq. 14 gives results that are within the acceptable range of accuracy. Typical values of $\bar{\alpha}$ are given in Table 1. For more precise work, such as the use of the platinum resistance thermometer to measure temperature (see Section 22-3), the linear approximation is not sufficient. In this case we can add terms in $(T - T_0)^2$ and $(T - T_0)^3$ to the right side of Eq. 14 to improve the precision. The coefficients of these additional terms must be determined empirically, in analogy with the coefficient $\bar{\alpha}$ in Eq. 14. ■

32-4 OHM'S LAW

Let us select a particular sample of conducting material, apply a uniform potential difference across it, and measure the resulting current. We repeat the measurement for various values of the potential difference and plot the results, as in Fig. 6*a*. The experimental points clearly fall along a straight line, which indicates that the ratio V/i (the inverse of the slope of the line) is a constant. The resistance of this device is a constant, independent of the potential difference across it or the current through it. Note that the line extends to negative potential differences and currents.

In this case, we say that the material obeys *Ohm's law*:

A conducting device obeys Ohm's law if the resistance between any pair of points is independent of the magnitude and polarity of the applied potential difference.

A material or a circuit element that obeys Ohm's law is called *ohmic*.

Modern electronic circuits also depend on devices that do *not* obey Ohm's law. An example of the current–voltage relationship for a nonohmic device (a *pn* junction diode) is shown in Fig. 6*b*. Note that the current does not increase linearly with the voltage, and also note that the device behaves very differently for negative potential differences than it does for positive ones.

We stress that the relationship $V = iR$ is *not* a statement of Ohm's law. A conductor obeys Ohm's law only if its V versus i graph is linear, that is, if R is independent of V and i. The relationship $R = V/i$ remains as the general definition of the resistance of a conductor whether or not the conductor obeys Ohm's law.

The microscopic equivalent of the relationship $V = iR$ is Eq. 10, $\mathbf{E} = \rho \mathbf{j}$. A conducting material is said to obey Ohm's law if a plot of E versus j is linear, that is, if the resistivity ρ is independent of E and j. Ohm's law is a specific property of certain materials and is not a general law of electromagnetism, for example, like Gauss' law.

Analogy Between Current and Heat Flow *(Optional)*

A close analogy exists between the flow of charge established by a potential difference and the flow of heat established by a temperature difference. Consider a thin electrically conducting slab of thickness Δx and area A. Let a potential difference ΔV be maintained between opposing faces. The current i is given by Eqs. 8 ($i = V/R$) and 13 ($R = \rho L/A$), or

$$i = \frac{V_a - V_b}{R} = \frac{(V_a - V_b)A}{\rho L} = -\frac{(V_b - V_a)A}{\rho \Delta x}.$$

In the limiting case of a slab of thickness dx this becomes

$$i = -\rho^{-1} A \frac{dV}{dx}$$

or, replacing the inverse of the resistivity by the conductivity σ,

$$\frac{dq}{dt} = -\sigma A \frac{dV}{dx}. \tag{17}$$

The minus sign in Eq. 17 indicates that positive charge flows in the direction of decreasing V; that is, dq/dt is positive when dV/dx is negative.

The analogous heat flow equation (see Section 25-7) is

$$\frac{dQ}{dt} = -kA \frac{dT}{dx}, \tag{18}$$

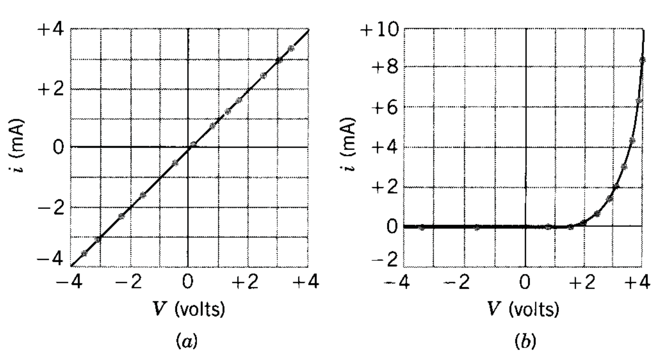

Figure 6 (*a*) A current–voltage plot for a material that obeys Ohm's law, in this case a 1000-Ω resistor. (*b*) A current–voltage plot for a material that does not obey Ohm's law, in this case a *pn* junction diode.

which shows that k, the thermal conductivity, corresponds to σ, and dT/dx, the temperature gradient, corresponds to dV/dx, the potential gradient. For pure metals there is more than a formal mathematical analogy between Eqs. 17 and 18. Both heat energy and charge are carried by the free electrons in such metals; empirically, a good electrical conductor (silver, say) is also a good heat conductor, and the electrical conductivity σ is directly related to the thermal conductivity k. ■

32-5 OHM'S LAW: A MICROSCOPIC VIEW

As we discussed previously, Ohm's law is not a fundamental law of electromagnetism because it depends on the properties of the conducting medium. The law is very simple in form, and it is curious that many materials obey it so well, whereas other materials do not obey it at all. Let us see if we can understand why metals obey Ohm's law, which we shall write (see Eq. 10) in the microscopic form $\mathbf{E} = \rho\mathbf{j}$.

In a metal the valence electrons are not attached to individual atoms but are free to move about within the lattice and are called *conduction electrons*. In copper there is one such electron per atom, the other 28 remaining bound to the copper nuclei to form ionic cores.

The theory of electrical conduction in metals is often based on the *free-electron model*, in which (as a first approximation) the conduction electrons are assumed to move freely throughout the conducting material, somewhat like molecules of gas in a container. In fact, the assembly of conduction electrons is sometimes called an *electron gas*. As we shall see, however, we cannot neglect the effect of the ion cores on this "gas."

The classical Maxwellian velocity distribution (see Section 24-3) for the electron gas would suggest that the conduction electrons have a broad distribution of velocities from zero to infinity, with a well-defined average. However, in considering the electrons we cannot ignore quantum mechanics, which gives a very different view. In the quantum distribution (see Fig. 16 of Chapter 24) the electrons that readily contribute to electrical conduction are concentrated in a very narrow interval of kinetic energies and therefore of speeds. To a very good approximation, we can assume that the electrons move with a uniform average speed. In the case of copper, this speed is about $\bar{v} = 1.6 \times 10^6$ m/s. Furthermore, whereas the Maxwellian average speed depends strongly on the temperature, the effective speed obtained from the quantum distribution is nearly independent of temperature.

In the absence of an electric field, the electrons move randomly, again like the molecules of gas in a container. Occasionally, an electron collides with an ionic core of the lattice, suffering a sudden change in direction in the process. As we did in the case of collisions of gas molecules, we can associate a mean free path λ and a mean free time τ to the average distance and time between collisions. (Collisions between the electrons themselves are rare and do not affect the electrical properties of the conductor.)

In an ideal metallic crystal (containing no defects or impurities) at 0 K, electron–lattice collisions would not occur, according to the predictions of quantum physics; that is, $\lambda \to \infty$ as $T \to 0$ K for ideal crystals. Collisions take place in actual crystals because (1) the ionic cores at any temperature T are vibrating about their equilibrium positions in a random way; (2) impurities, that is, foreign atoms, may be present; and (3) the crystal may contain lattice imperfections, such as missing atoms and displaced atoms. Consequently, the resistivity of a metal can be increased by (1) raising its temperature, (2) adding small amounts of impurities, and (3) straining it severely, as by drawing it through a die, to increase the number of lattice imperfections.

When we apply an electric field to a metal, the electrons modify their random motion in such a way that they drift slowly, in the opposite direction to that of the field, with an average drift speed v_d. This drift speed is very much less (by a factor of something like 10^{10}; see Sample Problem 2) than the effective average speed $\bar{v}$. Figure 7 suggests the relationship between these two speeds. The solid lines suggest a possible random path followed by an electron in the absence of an applied field; the electron proceeds from x to y, making six collisions on the way. The dashed lines show how this same event *might* have occurred if an electric field $\mathbf{E}$ had been applied. Note that the electron drifts steadily to the right, ending at y' rather than at y. In preparing Fig. 7, it has been assumed that the drift speed v_d is $0.02\bar{v}$; actually, it is more like $10^{-10}\bar{v}$, so that the "drift" exhibited in the figure is greatly exaggerated.

We can calculate the drift speed v_d in terms of the applied electric field E and of $\bar{v}$ and λ. When a field is applied to an electron in the metal, it experiences a force eE,

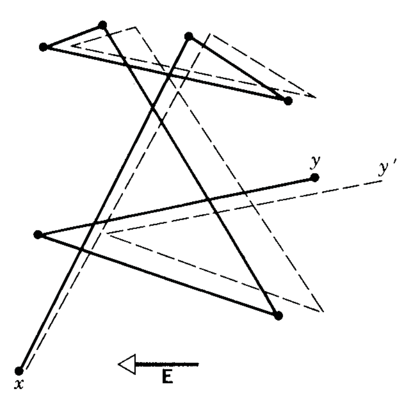

Figure 7 The solid line segments show an electron moving from x to y, making six collisions en route. The dashed lines show what its path *might* have been in the presence of an applied electric field $\mathbf{E}$. Note the gradual but steady drift in the direction of $-\mathbf{E}$. (Actually, the dashed lines should be slightly curved to represent the parabolic paths followed by the electrons between collisions.)

which imparts to it an acceleration a given by Newton's second law,

$$a = \frac{eE}{m}.$$

Consider an electron that has just collided with an ion core. The collision, in general, momentarily destroys the tendency to drift, and the electron has a truly random direction after the collision. During the time interval to the next collision, the electron's speed changes, on the average, by an amount $a(\lambda/\bar{v})$ or $a\tau$, where τ is the mean time between collisions. We identify this with the drift speed v_d, or*

$$v_d = a\tau = \frac{eE\tau}{m}. \tag{19}$$

We may also express v_d in terms of the current density (Eq. 6), which gives

$$v_d = \frac{j}{ne} = \frac{eE\tau}{m}.$$

Combining this with Eq. 9 ($\rho = E/j$), we finally obtain

$$\rho = \frac{m}{ne^2\tau}. \tag{20}$$

Note that m, n, and e in this equation are constants. Thus Eq. 20 can be taken as a statement that metals obey Ohm's law if we can show that τ is a constant. In particular, we must show that τ does not depend on the applied electric field E. In this case ρ does not depend on E, which (see Section 32-4) is the criterion that a material obey Ohm's law. The quantity τ depends on the speed distribution of the conduction electrons. We have seen that this distribution is affected only very slightly by the application of even a relatively large electric field, since $\bar{v}$ is of the order of 10^6 m/s, and v_d (see Sample Problem 2) is only of the order of 10^{-4} m/s, a ratio of 10^{10}. Whatever the value of τ is (for copper at 20°C, say) in the absence of a field, it remains essentially unchanged when the field is applied. Thus the right side of Eq. 20 is independent of E (which means that ρ is independent of E), and the material obeys Ohm's law.

* It may be tempting to write Eq. 19 as $v_d = \frac{1}{2}a\tau$, reasoning that $a\tau$ is the electron's *final* velocity, and thus that its *average* velocity is half that value. The extra factor of $\frac{1}{2}$ would be correct if we followed a typical electron, taking its drift speed to be the average of its velocity over its mean time τ between collisions. However, the drift speed is proportional to the current density j and must be calculated from the average velocity of *all* the electrons taken at one instant of time. For each electron, the velocity at any time is at, where t is the time since the last collision for that electron. Since the acceleration a is the same for all electrons, the average value of at at a given instant is $a\tau$, where τ is the average time since the last collision, which is the same as the mean time between collisions. For a discussion of this point, see *Electricity and Magnetism*, 2nd ed., by Edward Purcell (McGraw-Hill, 1985), Section 4.4. See also "Drift Speed and Collision Time," by Donald E. Tilley, *American Journal of Physics*, June 1976, p. 597.

Sample Problem 5 (*a*) What is the mean free time τ between collisions for the conduction electrons in copper? (*b*) What is the mean free path λ for these collisions? Assume an effective speed $\bar{v}$ of 1.6×10^6 m/s.

Solution (*a*) From Eq. 20 we have

$$\tau = \frac{m}{ne^2\rho}$$
$$= \frac{9.11 \times 10^{-31}\text{ kg}}{(8.49 \times 10^{28}\text{ m}^{-3})(1.60 \times 10^{-19}\text{ C})^2(1.69 \times 10^{-8}\ \Omega\cdot\text{m})}$$
$$= 2.48 \times 10^{-14}\text{ s}.$$

The value of n, the number of conduction electrons per unit volume in copper, was obtained from Sample Problem 2; the value of ρ comes from Table 1.

(*b*) We define the mean free path from

$$\lambda = \tau\bar{v} = (2.48 \times 10^{-14}\text{ s})(1.6 \times 10^6\text{ m/s})$$
$$= 4.0 \times 10^{-8}\text{ m} = 40\text{ nm}.$$

This is about 150 times the distance between nearest-neighbor ions in a copper lattice. A full treatment based on quantum physics reveals that we cannot view a "collision" as a direct interaction between an electron and an ion. Rather, it is an interaction between an electron and the thermal vibrations of the lattice, lattice imperfections, or lattice impurity atoms. An electron can pass very freely through an "ideal" lattice, that is, a geometrically "perfect" lattice close to the absolute zero of temperature. Mean free paths as large as 10 cm have been observed under such conditions.

32-6 ENERGY TRANSFERS IN AN ELECTRIC CIRCUIT

Figure 8 shows a circuit consisting of a battery B connected to a "black box." A steady current i exists in the connecting wires, and a steady potential difference V_{ab}

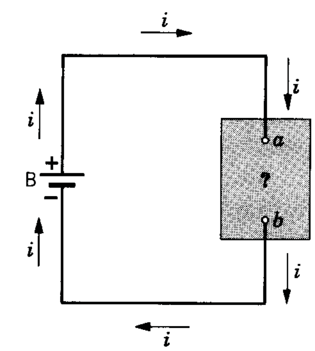

Figure 8 A battery B sets up a current i in a circuit containing a "black box," that is, a box whose contents are unknown.

exists between the terminals a and b. The box might contain a resistor, a motor, or a storage battery, among other things.

Terminal a, connected to the positive battery terminal, is at a higher potential than terminal b. The potential energy of a charge dq that moves through the box from a to b decreases by $dq\, V_{ab}$ (see Section 30-3). The conservation-of-energy principle tells us that this energy is transferred in the box from electric energy to some other form. What that other form will be depends on what is in the box. In a time dt the energy dU transferred inside the box is then

$$dU = dq\, V_{ab} = i\, dt\, V_{ab}.$$

We find the *rate* of energy transfer or power P according to

$$P = \frac{dU}{dt} = iV_{ab}. \qquad (21)$$

If the device in the box is a motor, the energy appears largely as mechanical work done by the motor; if the device is a storage battery that is being charged, the energy appears largely as stored chemical energy in this second battery.

If the device is a resistor, the energy appears in the resistor as internal energy (associated with atomic motion and observed, perhaps, as an increase in temperature). To see this, consider a stone of mass m that falls through a height h. It decreases its gravitational potential energy by mgh. If the stone falls in a vacuum or—for practical purposes—in air, this energy is transformed into kinetic energy of the stone. If the stone falls into the depths of the ocean, however, its speed eventually becomes constant, which means that the kinetic energy no longer increases. The potential energy that is steadily being made available as the stone falls then appears as internal energy in the stone and the surrounding water. It is the viscous, frictionlike drag of the water on the surface of the stone that stops the stone from accelerating, and it is at this surface that the transformation to internal energy occurs.

The course of an electron through the resistor is much like that of the stone through water. On average, the electrons travel with a constant drift speed v_d and thus do not gain kinetic energy. They lose electric energy through collisions with atoms of the resistor. As a result, the amplitudes of the atomic vibrations increase; on a macroscopic scale this can correspond to a temperature increase. Subsequently, there can be a flow of energy out of the resistor as heat, if the environment is at a lower temperature than the resistor.

For a resistor we can combine Eqs. 8 ($R = V/i$) and 21 and obtain either

$$P = i^2R \qquad (22)$$

or

$$P = \frac{V^2}{R}. \qquad (23)$$

Note that Eq. 21 applies to electrical energy transfer of *all* kinds; Eqs. 22 and 23 apply only to the transfer of electrical energy to internal energy in a resistor. Equations 22 and 23 are known as *Joule's law*, and the corresponding energy transferred to the resistor or its surroundings is called *Joule heating*. This law is a particular way of writing the conservation-of-energy principle for the special case in which electrical energy is transferred into internal energy in a resistor.

The unit of power that follows from Eq. 21 is the volt · ampere. We can show the volt · ampere to be equivalent to the watt as a unit of power by using the definitions of the volt (joule/coulomb) and ampere (coulomb/second):

$$1 \text{ volt}\cdot\text{ampere} = 1\ \frac{\text{joule}}{\text{coulomb}}\cdot\frac{\text{coulomb}}{\text{second}}$$
$$= 1\ \frac{\text{joule}}{\text{second}} = 1 \text{ watt}.$$

We previously introduced the watt as a unit of power in Section 7-5.

Sample Problem 6 You are given a length of heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. It is to be connected across a 120-V line. Under which circumstances will the wire dissipate more heat: (*a*) its entire length is connected across the line, or (*b*) the wire is cut in half and the two halves are connected in parallel across the line?

Solution (*a*) The power P dissipated by the entire wire is, from Eq. 23,

$$P = \frac{V^2}{R} = \frac{(120\ V)^2}{72\ \Omega} = 200\ \text{W}.$$

(*b*) The power for a wire of half length (and thus half resistance) is

$$P' = \frac{V^2}{\frac{1}{2}R} = \frac{(120\ V)^2}{36\ \Omega} = 400\ \text{W}.$$

There are two halves so that the power obtained from both of them is 800 W, or four times that for the single wire. This would seem to suggest that you could buy a heating wire, cut it in half, and reconnect it to obtain four times the heat output. Why is this not such a good idea?

32-7 SEMICONDUCTORS *(Optional)*

A class of materials called *semiconductors* is intermediate between conductors and insulators in its ability to conduct electricity. Among the elements, silicon and germaniun are common examples of room-temperature semiconductors. One important property of semiconductors is that their ability to conduct can be changed dramatically by external factors, such as by changes in the temperature, applied voltage, or incident light. You can see from Table 1 that, although pure silicon is a relatively poor conductor, a low concentration of impurity atoms (added to pure silicon at a level of one impurity atom per 10^6 silicon atoms) can change the conductivity by six or seven orders of magnitude. You can also see that the conductivity of silicon is at least an order of magnitude more sensitive to changes in temperature than that of a typical conductor. Because of these properties, semiconductors have found wide applications in such devices as switching and control circuits, and they are now essential components of integrated circuits and computer memories.

To describe the properties of conductors, insulators, and semiconductors in microscopic detail requires the application of the principles of quantum physics. However, we can gain a qualitative understanding of the differences between conductors, insulators, and semiconductors by referring to Fig. 9, which shows energy states that might typically represent electrons in conductors, semiconductors, and insulators. The electrons have permitted energies that are discrete or *quantized* (see Section 8-8), but which group together in *bands*. Within the bands, the permitted energy states, which are so close together that they are virtually continuous, may be *occupied* (electrons having the permitted energy) or *unoccupied* (no electrons having that energy). Between the bands there is an *energy gap*, which contains no states that an individual electron may occupy. An electron may jump from an occupied state to any unoccupied one. At ordinary temperatures, the internal energy distribution provides the source of the energy needed for electrons to jump to higher states.

Figure 9*a* illustrates the energy bands that represent a conductor. The valence band, which is the highest band occupied by electrons, is only partially occupied, so that electrons have many empty states to which they can easily jump. An applied electric field can encourage electrons to make these small jumps and contribute to a current in the material. This ease of movement of the electrons is what makes the material a conductor.

Figure 9*b* shows bands that might characterize a semiconductor, such as silicon. At very low temperature, the valence band is completely occupied, and the upper (conduction) band is completely empty. At ordinary temperatures, there is a small probability that an electron from one of the occupied states in the lower band has enough energy to jump across the gap to one of the empty states in the upper band. The probability for such a jump depends on the energy distribution, which according to Eq. 27 of Chapter 24 includes the factor $e^{-\Delta E/kT}$, where ΔE is the energy gap. Taking $\Delta E = 0.7$ eV (typical for silicon) and $kT = 0.025$ eV at room temperature, the exponential factor is 7×10^{-13}. Although this is a small number, there are so many electrons available in a piece of silicon (about 10^{23} per gram) that a reasonable number (perhaps 10^{11} per gram) are in the upper band. In this band they can move easily from occupied to empty states and contribute to the ability of a semiconductor to transport electric charge. (In the process of jumping to the conduction band, electrons leave vacancies or *holes* in the valence band. Other electrons in the valence band can jump to those vacancies, thereby also contributing to the conductivity.)

Another difference between conductors and semiconductors is in their temperature coefficients of resistivity. Metals are kept from being perfect conductors by deviations from the perfect lattice structure, such as might be caused by the presence of impurities or defects in the lattice. The vibration of the ion cores about their equilibrium lattice positions is a major contributor to the resistivity of metals. Since this effect increases with temperature, the resistivity of metals increases with temperature. The same effect of course also occurs in semiconductors, but it is overwhelmed by a much greater effect that *decreases* the resistivity with increasing temperature. As the temperature increases, more electrons acquire enough energy to be excited across the energy gap into the conduction band, thereby increasing the conductivity and decreasing the resistivity. As Table 1 shows, silicon (in contrast to the metals listed) has a *negative* temperature coefficient of resistivity.

Figure 9*c* shows typical energy bands for an insulator, such as sodium chloride. The band structure is very similar to that of a semiconductor, with the valence band occupied and the conduction band empty. The major difference is in the size of the energy gap, which might be typically 2 eV or more in the case of an insulator (compared with perhaps 0.7 eV in a semiconductor). This relatively small difference makes an enormous difference in the exponential factor that gives the probability of an electron acquiring enough energy to jump across the gap. For an insulator at room temperature, the factor $e^{-\Delta E/kT}$ is typically 2×10^{-35}, so that in a gram of material (10^{23} atoms) there is a

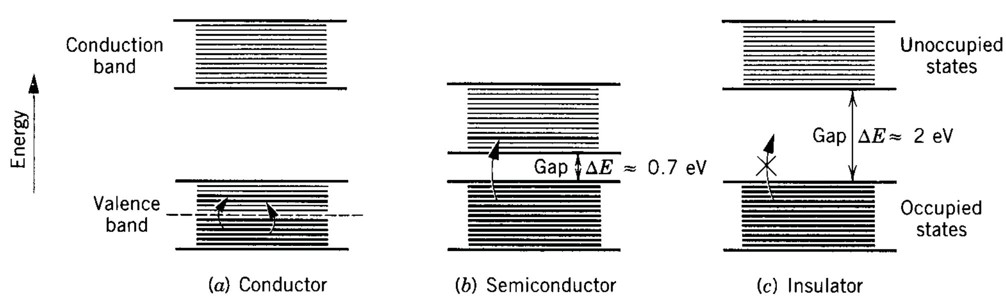

Figure 9 (*a*) Energy bands characteristic of a conductor. Below the dashed line, nearly all energy states are occupied, while nearly all states above that line are empty. Electrons can easily jump from occupied states to empty ones, as suggested by the arrows. (*b*) In a semiconductor, the dividing line between filled and empty states occurs in the gap. The electrical conductivity is determined in part by the number of electrons that jump to occupy states in the conduction band. (*c*) The energy bands in an insulator resemble those in a semiconductor; the major difference is in the size of the energy gap. At ordinary temperatures, there is no probability for an electron to jump to the empty states in the conduction band.

negligible probability at ordinary temperatures of *even one electron* being in the conduction band where it could move freely. In insulators, therefore, all electrons are confined to the valence band, where they have no empty states to enter and thus are not at all free to travel throughout the material.

Note that the principal difference between semiconductors and insulators is in the relationship between the gap energy and kT. At very low temperature, a semiconductor becomes an insulator, while at a high enough temperature (which is, however, above the point at which the material would be vaporized), an insulator could become a semiconductor.

We consider more details of the application of quantum theory to the structure of semiconductors in Chapter 53 of the extended text. ■

32-8 SUPERCONDUCTIVITY *(Optional)*

As we reduce the temperature of a conductor, the resistivity grows smaller, as Fig. 5 suggests. What happens as we approach the absolute zero of the temperature scale?

The part of the resistivity due to scattering of electrons by atoms vibrating from their equilibrium lattice positions decreases as the temperature decreases, because the amplitude of the vibration decreases with temperature. According to quantum theory, the atoms retain a certain minimum vibrational motion, even at the absolute zero of temperature. Furthermore, the contributions of defects and impurities to the resistivity remain as T falls to 0. We therefore expect the resistivity to decrease with decreasing temperature, but to remain finite at the lowest temperatures. Many materials do in fact show this type of behavior.

Quite a different kind of behavior was discovered in 1911 by the Dutch physicist Kammerlingh Onnes, who was studying the resistivity of mercury at low temperature. He discovered that, below a temperature of about 4 K, mercury suddenly lost all resistivity and became a *perfect* conductor, called a *superconductor.* This was not a gradual change, as Eq. 14 and Fig. 5 suggest, but a sudden transition, as indicated by Fig. 10. The resistivity of a superconductor is not just very small; it is zero! If a current is established in a superconducting material, it should persist forever, even with no electric field present.

The availability of superconducting materials immediately suggests a number of applications. (1) Energy can be transported and stored in electrical wires without resistive losses. That is, a power company can produce electrical energy when the demand is light, perhaps overnight, and store the current in a superconducting ring. Electrical power can then be released during peak demand times the following day. Such a ring now operates in Tacoma, Washington, to store 5 MW of power. In smaller laboratory test rings, currents have been stored for several years with no reduction. (2) Superconducting electromagnets can produce larger magnetic fields than conventional electromagnets. As we discuss in Chapter 35, a current-carrying wire gives rise to a magnetic field in the surrounding space, just as an electric charge sets up an electric field. With superconducting wires, larger currents and therefore larger magnetic fields can be produced. Applications of this technology include magnetically levitated trains and bending magnets for beams of particles in large accelerators such as Fermilab. (3) Superconducting components in electronic circuits would generate no Joule heating and would permit further miniaturization of circuits. The next generation of mainframe computers may employ superconducting components.

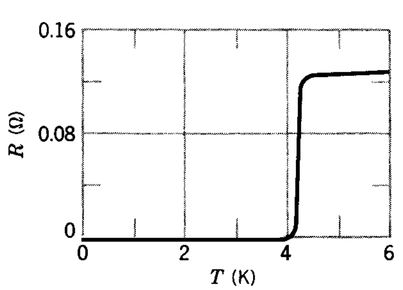

Figure 10 The resistivity of mercury drops to zero at a temperature of about 4 K. Mercury is a solid at this low temperature.

Progress in applying this exciting technology proceeded slowly in the 75 years following Kammerlingh Onnes' discovery for one reason: the elements and compounds that displayed superconductivity did so only at very low temperatures, in most cases below 20 K. To achieve such temperatures, the superconducting material is generally immersed in a bath of liquid helium at 4 K. The liquid helium is costly and so, while there have been many scientific applications of superconductivity, commercial applications have been held back by the high cost of liquid helium.

Beginning in 1986 a series of ceramic materials was discovered which remained superconducting at relatively high temperatures. The first of these kept its superconductivity to a temperature of 90 K. While this is still a low temperature by ordinary standards, it marks an important step: it can be maintained in a bath of liquid nitrogen (77 K), which costs about an order of magnitude less than liquid helium, thereby opening commercial possibilities that had not been feasible with liquid-helium-cooled materials.*

Superconductivity should not be regarded merely as an improvement in the conductivity of materials that are already good conductors. The best room-temperature conductors (copper, silver, and gold) do not show any superconductivity at all.

An understanding of this distinction can be found in the microscopic basis of superconductivity. Ordinary materials are good conductors if they have free electrons that can move easily through the lattice. Atoms of copper, silver, and gold have a single weakly bound valence electron that can be contributed to the electron gas that permeates the lattice. According to one theory, superconductors depend on the motion of highly correlated *pairs* of electrons. Since electrons generally don't like to form pairs, a special circumstance is required: two electrons each interact strongly with the lattice and thus in effect with each other. The situation is somewhat like two boats on a lake, where the wake left by the motion of one boat causes the other to move, even though the first boat did not exert a force directly on the second boat. Thus a good ordinary conductor depends on hav-

* See "The New Superconductors: Prospects for Applications," by Alan M. Wolsky, Robert F. Giese, and Edward J. Daniels, *Scientific American*, February 1989, p. 60, and "Superconductors Beyond 1-2-3," by Robert J. Cava, *Scientific American*, August 1990, p. 42.

ing electrons that interact *weakly* with the lattice, while a superconductor seems to require electrons that interact *strongly* with the lattice.

More details about superconductors and the application of quantum theory to understanding their properties can be found in Chapter 53 of the extended text. ■

QUESTIONS

1. Name other physical quantities that, like current, are scalars having a sense represented by an arrow in a diagram.

2. In our convention for the direction of current arrows (*a*) would it have been more convenient, or even possible, to have assumed all charge carriers to be negative? (*b*) Would it have been more convenient, or even possible, to have labeled the electron as positive, the proton as negative, and so on?

3. What experimental evidence can you give to show that the electric charges in current electricity and those in electrostatics are identical?

4. Explain in your own words why we can have $\mathbf{E} \neq 0$ inside a conductor in this chapter, whereas we took $\mathbf{E} = 0$ for granted in Section 29-4.

5. A current i enters one corner of a square sheet of copper and leaves at the opposite corner. Sketch arrows at various points within the square to represent the relative values of the current density $\mathbf{j}$. Intuitive guesses rather than detailed mathematical analyses are called for.

6. Can you see any logic behind the assignment of gauge numbers to household wire? See Problem 6. If not, then why is this system used?

7. A potential difference V is applied to a copper wire of diameter d and length L. What is the effect on the electron drift speed of (*a*) doubling V, (*b*) doubling L, and (*c*) doubling d?

8. Why is it not possible to measure the drift speed for electrons by timing their travel along a conductor?

9. Describe briefly some possible designs of variable resistors.

10. A potential difference V is applied to a circular cylinder of carbon by clamping it between circular copper electrodes, as in Fig. 11. Discuss the difficulty of calculating the resistance of the carbon cylinder using the relation $R = \rho L/A$.

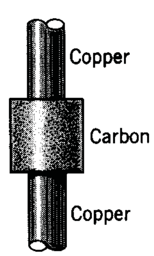

Figure 11 Question 10.

11. You are given a cube of aluminum and access to two battery terminals. How would you connect the terminals to the cube to ensure (*a*) a maximum and (*b*) a minimum resistance?

12. How would you measure the resistance of a pretzel-shaped block of metal? Give specific details to clarify the concept.

13. Sliding across the seat of an automobile can generate potentials of several thousand volts. Why isn't the slider electrocuted?

14. Discuss the difficulties of testing whether the filament of a light bulb obeys Ohm's law.

15. Will the drift velocity of electrons in a current-carrying metal conductor change when the temperature of the conductor is increased? Explain.

16. Explain why the momentum that conduction electrons transfer to the ions in a metal conductor does not give rise to a resultant force on the conductor.

17. List in tabular form similarities and differences between the flow of charge along a conductor, the flow of water through a horizontal pipe, and the conduction of heat through a slab. Consider such ideas as what causes the flow, what opposes it, what particles (if any) participate, and the units in which the flow may be measured.

18. How does the relation $V = iR$ apply to resistors that do *not* obey Ohm's law?

19. A cow and a man are standing in a meadow when lightning strikes the ground nearby. Why is the cow more likely to be killed than the man? The responsible phenomenon is called "step voltage."

20. The lines in Fig. 7 should be curved slightly. Why?

21. A fuse in an electrical circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. What are some characteristics of ideal fuse wire?

22. Why does an incandescent light bulb grow dimmer with use?

23. The character and quality of our daily lives are influenced greatly by devices that do not obey Ohm's law. What can you say in support of this claim?

24. From a student's paper: "The relationship $R = V/i$ tells us that the resistance of a conductor is directly proportional to the potential difference applied to it." What do you think of this proposition?

25. Carbon has a negative temperature coefficient of resistivity. This means that its resistivity drops as its temperature increases. Would its resistivity disappear entirely at some high enough temperature?

26. What special characteristics must heating wire have?

27. Equation 22 ($P = i^2R$) seems to suggest that the rate of increase of internal energy in a resistor is reduced if the resistance is made less; Eq. 23 ($P = V^2/R$) seems to suggest just the opposite. How do you reconcile this apparent paradox?

28. Why do electric power companies reduce voltage during times of heavy demand? What is being saved?

29. Is the filament resistance lower or higher in a 500-W light bulb than in a 100-W bulb? Both bulbs are designed to operate at 120 V.

30. Five wires of the same length and diameter are connected in turn between two points maintained at constant potential difference. Will internal energy be developed at a faster rate in the wire of (*a*) the smallest or (*b*) the largest resistance?

31. Why is it better to send 10 MW of electric power long distances at 10 kV rather than at 220 V?

PROBLEMS

Section 32-2 Current Density

1. A current of 4.82 A exists in a 12.4-Ω resistor for 4.60 min. (*a*) How much charge and (*b*) how many electrons pass through any cross section of the resistor in this time?
2. The current in the electron beam of a typical video display terminal is 200 μA. How many electrons strike the screen each minute?
3. Suppose that we have 2.10×10^8 doubly charged positive ions per cubic centimeter, all moving north with a speed of 1.40×10^5 m/s. (*a*) Calculate the current density, in magnitude and direction. (*b*) Can you calculate the total current in this ion beam? If not, what additional information is needed?
4. A small but measurable current of 123 pA exists in a copper wire whose diameter is 2.46 mm. Calculate (*a*) the current density and (*b*) the electron drift speed. See Sample Problem 2.
5. Suppose that the material composing a fuse (see Question 21) melts once the current density rises to 440 A/cm^2. What diameter of cylindrical wire should be used for the fuse to limit the current to 0.552 A?
6. The (United States) National Electric Code, which sets maximum safe currents for rubber-insulated copper wires of various diameters, is given (in part) below. Plot the safe current density as a function of diameter. Which wire gauge has the maximum safe current density?

Gauge[a]	4	6	8	10	12	14	16	18
Diameter (mils)[b]	204	162	129	102	81	64	51	40
Safe current (A)	70	50	35	25	20	15	6	3

[a] A way of identifying the wire diameter.
[b] 1 mil = 10^{-3} in.

7. A current is established in a gas discharge tube when a sufficiently high potential difference is applied across the two electrodes in the tube. The gas ionizes; electrons move toward the positive terminal and singly charged positive ions toward the negative terminal. What are the magnitude and direction of the current in a hydrogen discharge tube in which 3.1×10^{18} electrons and 1.1×10^{18} protons move past a cross-sectional area of the tube each second?
8. A *pn* junction is formed from two different semiconducting materials in the form of identical cylinders with radius 0.165 mm, as depicted in Fig. 12. In one application 3.50×10^{15} electrons per second flow across the junction from the *n* to the *p* side while 2.25×10^{15} holes per second flow from the *p* to the *n* side. (A hole acts like a particle with charge $+1.6 \times 10^{-19}$ C.) Find (*a*) the total current and (*b*) the current density.

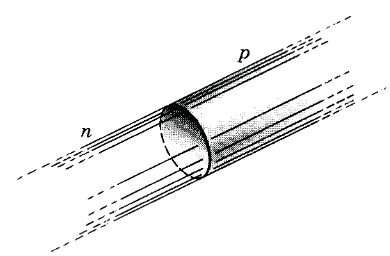

Figure 12 Problem 8.

9. You are given an isolated conducting sphere of 13-cm radius. One wire carries a current of 1.0000020 A into it. Another wire carries a current of 1.0000000 A out of it. How long would it take for the sphere to increase in potential by 980 V?
10. The belt of an electrostatic accelerator is 52.0 cm wide and travels at 28.0 m/s. The belt carries charge into the sphere at a rate corresponding to 95.0 μA. Compute the surface charge density on the belt. See Section 30-11.
11. Near the Earth, the density of protons in the solar wind is 8.70 cm^{-3} and their speed is 470 km/s. (*a*) Find the current density of these protons. (*b*) If the Earth's magnetic field did not deflect them, the protons would strike the Earth. What total current would the Earth receive?
12. In a hypothetical fusion research lab, high-temperature helium gas is completely ionized, each helium atom being separated into two free electrons and the remaining positively charged nucleus (alpha particle). An applied electric field causes the alpha particles to drift to the east at 25 m/s while the electrons drift to the west at 88 m/s. The alpha particle density is 2.8×10^{15} cm^{-3}. Calculate the net current density; specify the current direction.
13. How long does it take electrons to get from a car battery to the starting motor? Assume that the current is 115 A and the electrons travel through copper wire with cross-sectional area 31.2 mm^2 and length 85.5 cm. See Sample Problem 2.
14. A steady beam of alpha particles ($q = 2e$) traveling with kinetic energy 22.4 MeV carries a current of 250 nA. (*a*) If the beam is directed perpendicular to a plane surface, how many alpha particles strike the surface in 2.90 s? (*b*) At any instant, how many alpha particles are there in a given 18.0-cm length of the beam? (*c*) Through what potential difference was it necessary to accelerate each alpha particle from rest to bring it to an energy of 22.4 MeV?
15. In the two intersecting storage rings of circumference 950 m at CERN, protons of kinetic energy 28.0 GeV form beams of current 30.0 A each. (*a*) Find the total charge carried by the

protons in each ring. Assume that the protons travel at the speed of light. (*b*) A beam is deflected out of a ring onto a 43.5-kg copper block. By how much does the temperature of the block rise?

16. (*a*) The current density across a cylindrical conductor of radius R varies according to the equation

$$j = j_0(1 - r/R),$$

where r is the distance from the axis. Thus the current density is a maximum j_0 at the axis $r = 0$ and decreases linearly to zero at the surface $r = R$. Calculate the current in terms of j_0 and the conductor's cross-sectional area $A = \pi R^2$. (*b*) Suppose that, instead, the current density is a maximum j_0 at the surface and decreases linearly to zero at the axis, so that

$$j = j_0 r/R.$$

Calculate the current. Why is the result different from (*a*)?

Section 32-3 Resistance, Resistivity, and Conductivity

17. A steel trolley-car rail has a cross-sectional area of 56 cm^2. What is the resistance of 11 km of rail? The resistivity of the steel is $3.0 \times 10^{-7}\ \Omega\cdot m$.

18. A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with two conductors being held one in each hand. If the electrician's resistance is 1800 Ω, what might the fatal voltage be? (Electricians often work with "live" wires.)

19. A wire 4.0 m long and 6.0 mm in diameter has a resistance of 15 mΩ. A potential difference of 23 V is applied between the ends. (*a*) What is the current in the wire? (*b*) Calculate the current density. (*c*) Calculate the resistivity of the wire material. Can you identify the material? See Table 1.

20. A fluid with resistivity 9.40 Ω·m seeps into the space between the plates of a 110-pF parallel-plate air capacitor. When the space is completely filled, what is the resistance between the plates?

21. Show that if changes in the dimensions of a conductor with changing temperature can be ignored, then the resistance varies with temperature according to $R - R_0 = \bar{\alpha} R_0(T - T_0)$.

22. From the slope of the line in Fig. 5, estimate the average temperature coefficient of resistivity for copper at room temperature and compare with the value given in Table 1.

23. (*a*) At what temperature would the resistance of a copper conductor be double its resistance at 20°C? (Use 20°C as the reference point in Eq. 14; compare your answer with Fig. 5.) (*b*) Does this same temperature hold for all copper conductors, regardless of shape or size?

24. The copper windings of a motor have a resistance of 50 Ω at 20°C when the motor is idle. After running for several hours the resistance rises to 58 Ω. What is the temperature of the windings? Ignore changes in the dimensions of the windings. See Table 1.

25. A 4.0-cm-long caterpillar crawls in the direction of electron drift along a 5.2-mm-diameter bare copper wire that carries a current of 12 A. (*a*) Find the potential difference between the two ends of the caterpillar. (*b*) Is its tail positive or negative compared to its head? (*c*) How much time could it take the caterpillar to crawl 1.0 cm and still keep up with the drifting electrons in the wire?

26. A coil is formed by winding 250 turns of insulated gauge 8 copper wire (see Problem 6) in a single layer on a cylindrical form whose radius is 12.2 cm. Find the resistance of the coil. Neglect the thickness of the insulation. See Table 1.

27. A wire with a resistance of 6.0 Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are not changed during the drawing process.

28. What must be the diameter of an iron wire if it is to have the same resistance as a copper wire 1.19 mm in diameter, both wires being the same length?

29. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter D. Conductor B is a hollow tube of outside diameter $2D$ and inside diameter D. Find the resistance ratio, R_A/R_B, measured between their ends.

30. A copper wire and an iron wire of the same length have the same potential difference applied to them. (*a*) What must be the ratio of their radii if the current is to be the same? (*b*) Can the current density be made the same by suitable choices of the radii?

31. An electrical cable consists of 125 strands of fine wire, each having 2.65-μΩ resistance. The same potential difference is applied between the ends of each strand and results in a total current of 750 mA. (*a*) What is the current in each strand? (*b*) What is the applied potential difference? (*c*) What is the resistance of the cable?

32. A common flashlight bulb is rated at 310 mA and 2.90 V, the values of the current and voltage under operating conditions. If the resistance of the bulb filament when cold (T_0 = 20°C) is 1.12 Ω, calculate the temperature of the filament when the bulb is on. The filament is made of tungsten. Assume that Eq. 14 holds over the temperature range encountered.

33. When 115 V is applied across a 9.66-m-long wire, the current density is 1.42 A/cm^2. Calculate the conductivity of the wire material.

34. A block in the shape of a rectangular solid has a cross-sectional area of 3.50 cm^2, a length of 15.8 cm, and a resistance of 935 Ω. The material of which the block is made has 5.33×10^{22} conduction electrons/m^3. A potential difference of 35.8 V is maintained between its ends. (*a*) Find the current in the block. (*b*) Assuming that the current density is uniform, what is its value? Calculate (*c*) the drift velocity of the conduction electrons and (*d*) the electric field in the block.

35. Copper and aluminum are being considered for a high-voltage transmission line that must carry a current of 62.3 A. The resistance per unit length is to be 0.152 Ω/km. Compute for each choice of cable material (*a*) the current density and (*b*) the mass of 1.00 m of the cable. The densities of copper and aluminum are 8960 and 2700 kg/m^3, respectively.

36. In the lower atmosphere of the Earth there are negative and positive ions, created by radioactive elements in the soil and

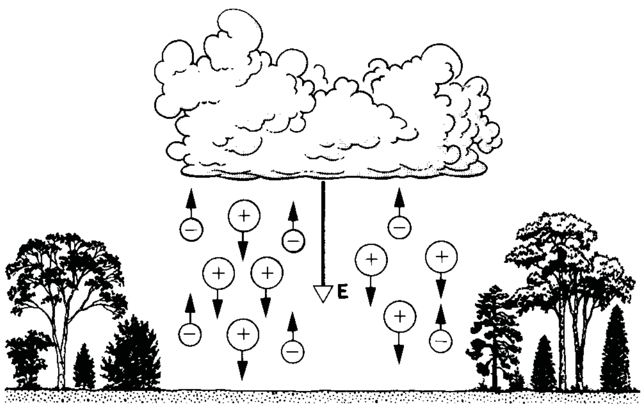

Figure 13 Problem 36.

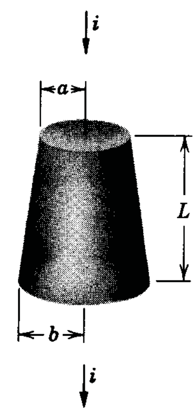

Figure 14 Problem 40.

cosmic rays from space. In a certain region, the atmospheric electric field strength is 120 V/m, directed vertically down. Due to this field, singly charged positive ions, 620 per cm^3, drift downward, and singly charged negative ions, 550 per cm^3, drift upward; see Fig. 13. The measured conductivity is $2.70 \times 10^{-14}/\Omega \cdot m$. Calculate (*a*) the ion drift speed, assumed the same for positive and negative ions, and (*b*) the current density.

37. A rod of a certain metal is 1.6 m long and 5.5 mm in diameter. The resistance between its ends (at 20°C) is $1.09 \times 10^{-3}\ \Omega$. A round disk is formed of this same material, 2.14 cm in diameter and 1.35 mm thick. (*a*) What is the material? (*b*) What is the resistance between the opposing round faces, assuming equipotential sufaces?

38. When a metal rod is heated, not only its resistance but also its length and its cross-sectional area change. The relation $R = \rho L/A$ suggests that all three factors should be taken into account in measuring ρ at various temperatures. (*a*) If the temperature changes by 1.0 C°, what fractional changes in R, L, and A occur for a copper conductor? (*b*) What conclusion do you draw? The coefficient of linear expansion is $1.7 \times 10^{-5}/C°$.

39. It is desired to make a long cylindrical conductor whose temperature coefficient of resistivity at 20°C will be close to zero. If such a conductor is made by assembling alternate disks of iron and carbon, find the ratio of the thickness of a carbon disk to that of an iron disk. (For carbon, $\rho = 3500 \times 10^{-8}\ \Omega \cdot m$ and $\alpha = -0.50 \times 10^{-3}/C°$.)

40. A resistor is in the shape of a truncated right circular cone (Fig. 14). The end radii are a and b, and the altitude is L. If the taper is small, we may assume that the current density is uniform across any cross section. (*a*) Calculate the resistance of this object. (*b*) Show that your answer reduces to $\rho L/A$ for the special case of zero taper ($a = b$).

Section 32-4 Ohm's Law

41. For a hypothetical electronic device, the potential difference V in volts, measured across the device, is related to the current i in mA by $V = 3.55\, i^2$. (*a*) Find the resistance when the current is 2.40 mA. (*b*) At what value of the current is the resistance equal to 16.0 Ω?

42. Using data from Fig. 6*b*, plot the resistance of the *pn* junction diode as a function of applied potential difference.

Section 32-5 Ohm's Law: A Microscopic View

43. Calculate the mean free time between collisions for conduction electrons in aluminum at 20°C. Each atom of aluminum contributes three conduction electrons. Take needed data from Table 1 and Appendix D. See also Sample Problem 2.

44. Show that, according to the free-electron model of electrical conduction in metals and classical physics, the resistivity of metals should be proportional to $\sqrt{T}$, where T is absolute temperature. (*Hint*: Treat the electrons as an ideal gas.)

Section 32-6 Energy Transfers in an Electric Circuit

45. A student's 9.0-V, 7.5-W portable radio was left on from 9:00 p.m. until 3:00 a.m. How much charge passed through the wires?

46. The headlights of a moving car draw 9.7 A from the 12-V alternator, which is driven by the engine. Assume the alternator is 82% efficient and calculate the horsepower the engine must supply to run the lights.

47. A space heater, operating from a 120-V line, has a hot resistance of 14.0 Ω. (*a*) At what rate is electrical energy transfered into internal energy? (*b*) At 5.22¢/kW·h, what does it cost to operate the device for 6 h 25 min?

48. The National Board of Fire Underwriters has fixed safe current-carrying capacities for various sizes and types of wire. For #10 rubber-coated copper wire (diameter = 0.10 in.) the maximum safe current is 25 A. At this current, find (*a*) the current density, (*b*) the electric field, (*c*) the potential difference for 1000 ft of wire, and (*d*) the rate at which internal energy is developed for 1000 ft of wire.

49. A 100-W light bulb is plugged into a standard 120-V outlet. (*a*) How much does it cost per month (31 days) to leave the light turned on? Assume electric energy cost 6¢/kW·h. (*b*) What is the resistance of the bulb? (*c*) What is the current in the bulb? (*d*) Is the resistance different when the bulb is turned off?

50. A Nichrome heater dissipates 500 W when the applied potential difference is 110 V and the wire temperature is 800°C. How much power would it dissipate if the wire temperature were held at 200°C by immersion in a bath of cooling oil? The applied potential difference remains the same; α for Nichrome at 800°C is 4.0×10^{-4}/C°.

51. An electron linear accelerator produces a pulsed beam of electrons. The pulse current is 485 mA and the pulse duration is 95.0 ns. (*a*) How many electrons are accelerated per pulse? (*b*) Find the average current for a machine operating at 520 pulses/s. (*c*) If the electrons are accelerated to an energy of 47.7 MeV, what are the values of average and peak power outputs of the accelerator?

52. A cylindrical resistor of radius 5.12 mm and length 1.96 cm is made of material that has a resistivity of 3.50×10^{-5} $\Omega \cdot$m. What are (*a*) the current density and (*b*) the potential difference when the power dissipation is 1.55 W?

53. A heating element is made by maintaining a potential difference of 75 V along the length of a Nichrome wire with a 2.6 mm^2 cross section and a resistivity of 5.0×10^{-7} $\Omega \cdot$m. (*a*) If the element dissipates 4.8 kW, what is its length? (*b*) If a potential difference of 110 V is used to obtain the same power output, what should the length be?

54. A coil of current-carrying Nichrome wire is immersed in a liquid contained in a calorimeter. When the potential difference across the coil is 12 V and the current through the coil is 5.2 A, the liquid boils at a steady rate, evaporating at the rate of 21 mg/s. Calculate the heat of vaporization of the liquid.

55. A resistance coil, wired to an external battery, is placed inside an adiabatic cylinder fitted with a frictionless piston and containing an ideal gas. A current $i = 240$ mA flows through the coil, which has a resistance $R = 550\ \Omega$. At what speed v must the piston, mass $m = 11.8$ kg, move upward in order that the temperature of the gas remains unchanged? See Fig. 15.

56. An electric immersion heater normally takes 93.5 min to bring cold water in a well-insulated container to a certain temperature, after which a thermostat switches the heater off. One day the line voltage is reduced by 6.20% because of a laboratory overload. How long will it now take to heat the water? Assume that the resistance of the heating element is the same for each of these two modes of operation.

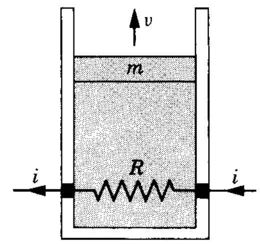

Figure 15 Problem 55.

57. Two isolated conducting spheres, each of radius 14.0 cm, are charged to potentials of 240 and 440 V and are then connected by a fine wire. Calculate the internal energy developed in the wire.

58. The current carried by the electron beam in a particular cathode ray tube is 4.14 mA. The speed of the electrons is 2.82×10^7 m/s and the beam travels a distance of 31.5 cm in reaching the screen. (*a*) How many electrons are in the beam at any instant? (*b*) Find the power dissipated at the screen. (Ignore relativistic effects.)

59. A 420-W immersion heater is placed in a pot containing 2.10 liters of water at 18.5°C. (*a*) How long will it take to bring the water to boiling temperature, assuming that 77.0% of the available energy is absorbed by the water? (*b*) How much longer will it take to boil half the water away?

60. A 32-μF capacitor is connected across a programmed power supply. During the interval from $t = 0$ to $t = 3$ s the output voltage of the supply is given by $V(t) = 6 + 4t - 2t^2$ volts. At $t = 0.50$ s find (*a*) the charge on the capacitor, (*b*) the current into the capacitor, and (*c*) the power output from the power supply.

61. A potential difference V is applied to a wire of cross-sectional area A, length L, and conductivity σ. You want to change the applied potential difference and draw out the wire so the power dissipated is increased by a factor of 30 and the current is increased by a factor of 4. What should be the new values of the (*a*) length and (*b*) cross-sectional area?