Universidad Nacional Autónoma de México
Facultad de Química
Departamento de Ingeniería Metalúrgica
Introducción a la Ingeniería de Procesos Metalúrgicos y de Materiales
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Tablas y Ecuaciones de Dinámica de Fluidos

Componentes del tensor de esfuerzos para fluidos newtonianos en coordenadas cartesianas[1] (x, y, z):

$$\bar{\tau}_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla \cdot \bar{v})\right]$$

$$\bar{\tau}_{yy} = -\mu\left[2\frac{\partial v_y}{\partial y} - \frac{2}{3}(\nabla \cdot \bar{v})\right]$$

$$\bar{\tau}_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \bar{v})\right]$$

$$\bar{\tau}_{xy} = \bar{\tau}_{yx} = -\mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$

$$\bar{\tau}_{xz} = \bar{\tau}_{zx} = -\mu\left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]$$

$$\bar{\tau}_{yz} = \bar{\tau}_{zy} = -\mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$

[1] $\nabla \cdot \bar{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Componentes del tensor de esfuerzos para fluidos newtonianos en coordenadas cilíndricas[2] (r, θ, z):

$$\bar{\tau}_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla\cdot\bar{v})\right]$$

$$\bar{\tau}_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla\cdot\bar{v})\right]$$

$$\bar{\tau}_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla\cdot\bar{v})\right]$$

$$\bar{\tau}_{r\theta} = \bar{\tau}_{\theta r} = -\mu\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$\bar{\tau}_{rz} = \bar{\tau}_{zr} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]$$

$$\bar{\tau}_{\theta z} = \bar{\tau}_{z\theta} = -\mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

[2] $\nabla\cdot\bar{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

Componentes del tensor de esfuerzos para fluidos newtonianos en coordenadas esféricas[3] (r, θ, φ):

$$\bar{\tau}_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla\cdot\bar{v})\right]$$

$$\bar{\tau}_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla\cdot\bar{v})\right]$$

$$\bar{\tau}_{\varphi\varphi} = -\mu\left[2\left(\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right) - \frac{2}{3}(\nabla\cdot\bar{v})\right]$$

$$\bar{\tau}_{r\theta} = \bar{\tau}_{\theta r} = -\mu\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$\bar{\tau}_{r\varphi} = \bar{\tau}_{\varphi r} = -\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \varphi} + r\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right)\right]$$

$$\bar{\tau}_{\theta\varphi} = \bar{\tau}_{\varphi\theta} = -\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi} + \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\varphi}{\sin\theta}\right)\right]$$

[3] $\nabla\cdot\bar{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}$

CONSERVACIÓN DE MASA

Ecuación de continuidad (conservación de masa) en coordenadas cartesianas (x, y, z):

$$-\left[\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)\right]=\frac{\partial \rho}{\partial t}$$

Ecuación de continuidad (conservación de masa) en coordenadas cilíndricas (r, θ, z):

$$-\left[\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)\right]=\frac{\partial \rho}{\partial t}$$

Ecuación de continuidad (conservación de masa) en coordenadas esféricas (r, θ, φ):

$$-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(\rho v_\varphi)\right]=\frac{\partial \rho}{\partial t}$$

CONSERVACIÓN DE MOMENTUM

Ecuaciones de Navier – Stokes (conservación de momentum) en coordenadas cartesianas (x, y, z) en términos de $\bar{\bar{\tau}}$.

Componente x:

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} - \left(\frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z}\right) + \rho g_x$$

Componente y:

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y} - \left(\frac{\partial \bar{\tau}_{xy}}{\partial x} + \frac{\partial \bar{\tau}_{yy}}{\partial y} + \frac{\partial \bar{\tau}_{zy}}{\partial z}\right) + \rho g_y$$

Componente z:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} - \left(\frac{\partial \bar{\tau}_{xz}}{\partial x} + \frac{\partial \bar{\tau}_{yz}}{\partial y} + \frac{\partial \bar{\tau}_{zz}}{\partial z}\right) + \rho g_z$$

Ecuaciones de Navier – Stokes (conservación de momentum) en coordenadas cartesianas (x, y, z) para un fluido incompresible (ρ = cte) y newtoniano (μ = cte).

Componente x:

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

Componente y:

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

Componente z:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Ecuaciones de Navier – Stokes (conservación de momentum) en coordenadas cilíndricas (r, θ, z) en términos de $\bar{\bar{\tau}}$.

Componente r:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{{v_\theta}^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial P}{\partial r}-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\bar{\tau}_{rr})+\frac{1}{r}\frac{\partial \bar{\tau}_{r\theta}}{\partial \theta}-\frac{\bar{\tau}_{\theta\theta}}{r}+\frac{\partial \bar{\tau}_{rz}}{\partial z}\right)+\rho g_r$$

Componente θ:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\bar{\tau}_{r\theta})+\frac{1}{r}\frac{\partial \bar{\tau}_{\theta\theta}}{\partial \theta}+\frac{\partial \bar{\tau}_{z\theta}}{\partial z}\right)+\rho g_\theta$$

Componente z:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial P}{\partial z}-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\bar{\tau}_{rz})+\frac{1}{r}\frac{\partial \bar{\tau}_{\theta z}}{\partial \theta}+\frac{\partial \bar{\tau}_{zz}}{\partial z}\right)+\rho g_z$$

Ecuaciones de Navier – Stokes (conservación de momentum) en coordenadas cilíndricas (r, θ, z) para un fluido incompresible (ρ = cte) y newtoniano (μ = cte).

Componente r:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{{v_\theta}^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial P}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$$

Componente θ:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$$

Componente z:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

Ecuaciones de Navier – Stokes (conservación de momentum) en coordenadas esféricas (r, θ, φ) en términos de $\bar{\bar{\tau}}$.

Componente r:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\varphi}{r\sin\theta}\frac{\partial v_r}{\partial \varphi}-\frac{{v_\theta}^2+{v_\varphi}^2}{r}\right)=-\frac{\partial P}{\partial r}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\bar{\tau}_{rr})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\bar{\tau}_{r\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial\bar{\tau}_{r\varphi}}{\partial\varphi}-\frac{\bar{\tau}_{\theta\theta}+\bar{\tau}_{\varphi\varphi}}{r}\right)+\rho g_r$$

Componente θ:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\varphi}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi}+\frac{v_r v_\theta}{r}-\frac{{v_\varphi}^2\cot\theta}{r}\right)$$

$$=-\frac{1}{r}\frac{\partial P}{\partial \theta}-\left(\begin{array}{c}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\bar{\tau}_{r\theta})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\bar{\tau}_{\theta\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial\bar{\tau}_{\theta\varphi}}{\partial\varphi}\\-\frac{\bar{\tau}_{r\theta}}{r}-\frac{\cot\theta}{r}\bar{\tau}_{\varphi\varphi}\end{array}\right)+\rho g_\theta$$

Componente φ:

$$\rho\left(\frac{\partial v_\varphi}{\partial t}+v_r\frac{\partial v_\varphi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\varphi}{\partial \theta}+\frac{v_\varphi}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}+\frac{v_r v_\varphi}{r}-\frac{v_\theta v_\varphi\cot\theta}{r}\right)=-\frac{1}{r\sin\theta}\frac{\partial P}{\partial \varphi}-\left(\begin{array}{c}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\bar{\tau}_{r\varphi})+\frac{1}{r}\frac{\partial\bar{\tau}_{\theta\varphi}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\bar{\tau}_{\varphi\varphi}}{\partial\varphi}\\-\frac{\bar{\tau}_{r\varphi}}{r}+\frac{2\cot\theta}{r}\bar{\tau}_{\theta\varphi}\end{array}\right)+\rho g_\varphi$$

Ecuaciones de Navier – Stokes (conservación de momentum) en coordenadas esféricas (r, θ, φ) para un fluido incompresible (ρ = cte) y newtoniano (μ = cte)[4].

Componente r:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\varphi}{r\sin\theta}\frac{\partial v_r}{\partial \varphi}-\frac{{v_\theta}^2+{v_\varphi}^2}{r}\right)=-\frac{\partial P}{\partial r}+\mu\left[\nabla^2 v_r-\frac{2}{r^2}v_r-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}-\frac{2}{r^2}v_\theta\cot\theta-\frac{1}{r^2\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}\right]+\rho g_r$$

Componente θ:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\varphi}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi}+\frac{v_r v_\theta}{r}-\frac{{v_\varphi}^2\cot\theta}{r}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left[\nabla^2 v_\theta+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta}{r^2\,sin^2\,\theta}-\frac{2\cos\theta}{r^2\,sin^2\,\theta}\frac{\partial v_\varphi}{\partial \varphi}\right]+\rho g_\theta$$

Componente φ:

$$\rho\left(\frac{\partial v_\varphi}{\partial t}+v_r\frac{\partial v_\varphi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\varphi}{\partial \theta}+\frac{v_\varphi}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}+\frac{v_r v_\varphi}{r}-\frac{v_\theta v_\varphi\cot\theta}{r}\right)$$
$$=-\frac{1}{r\sin\theta}\frac{\partial P}{\partial \varphi}+\mu\left[\nabla^2 v_\varphi-\frac{v_\varphi}{r^2\,sin^2\,\theta}+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \varphi}+\frac{2\cos\theta}{r^2\,sin^2\,\theta}\frac{\partial v_\theta}{\partial \varphi}\right]+\rho g_\varphi$$

[4] $\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)+\frac{1}{r^2\,sin^2\,\theta}\left(\frac{\partial^2}{\partial \varphi^2}\right)$

Tabla.- Viscosidad de varios fluidos a diferentes temperaturas.

Fluido	**Temperatura (°C)**	**Viscosidad (cP)**	**Viscosidad (N s m^{-2})**
Hidrógeno	0	0.0084	8.4×10^{-6}
	20.7	0.0088	8.8
	229	0.0126	12.6
	490	0.0167	16.7
	825	0.0214	21.4
Aire	0	0.0171	17.1×10^{-6}
	18	0.0183	18.3
	229	0.0264	26.4
	409	0.0341	34.1
	810	0.0442	44.2
	1134	0.0521	52.2
Agua	0	1.79	17.9×10^{-3}
	20	1.01	10.1
	60	0.469	4.69
	100	0.284	2.84
Hierro	1550	6.7	6.7×10^{-3}
	1600	6.1	6.1
	1700	5.6	5.6
	1800	5.3	5.3
	1850	5.2	5.2

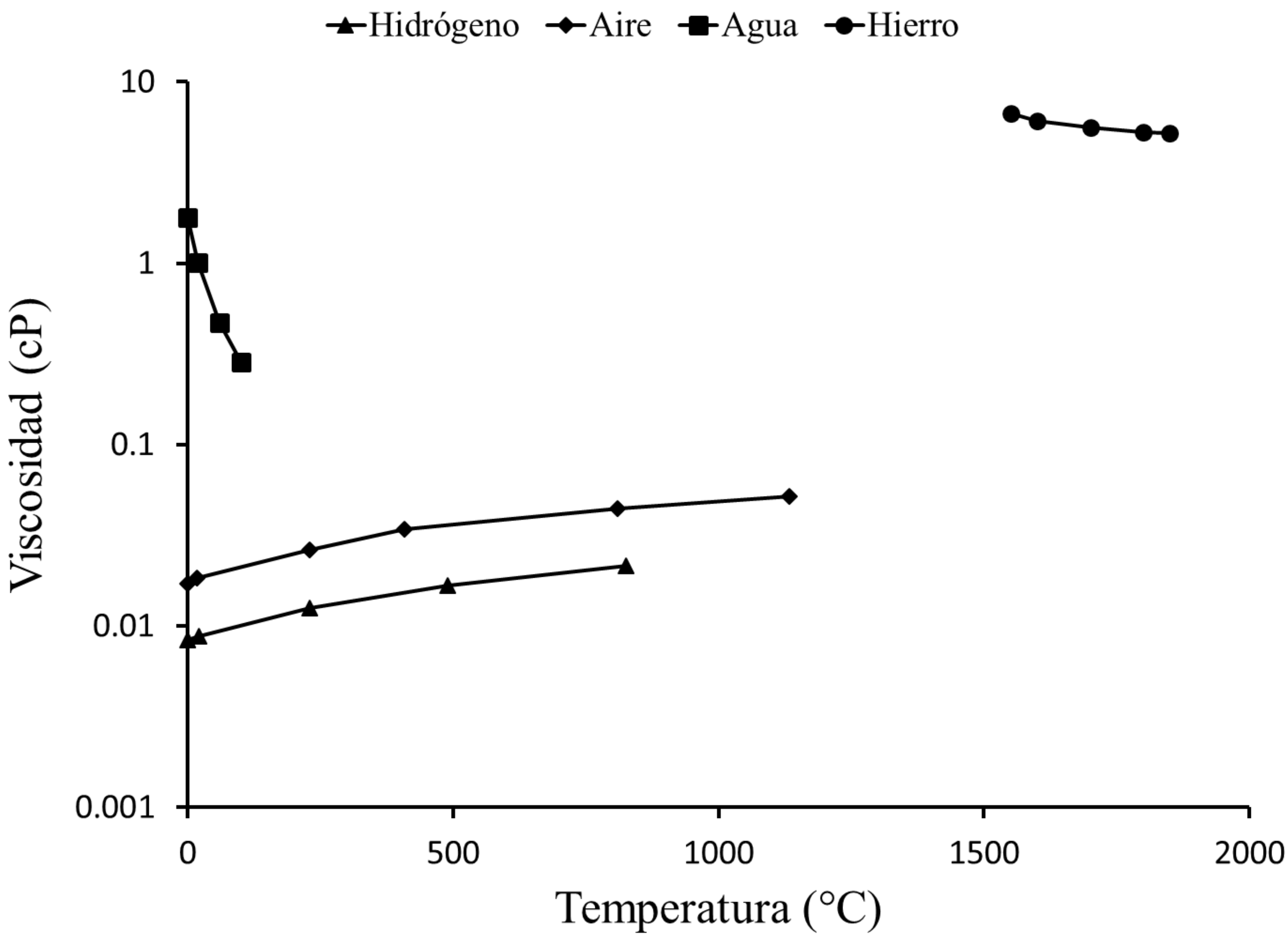

Figura.- Viscosidad de diversos fluidos en función de la temperatura.

Tabla.- Longitud equivalente para distintos accesorios de tubería.

Accesorio	**Diámetros de tubería**
Codo de 45°	15
Codo de 90° (radio estándar)	30 - 40
Codo de 90° (cuadrado)	60
Entrada a una conexicón en T desde un lateral	60
Entrada a una conexicón en T hacia un lateral	90
Uniones y coples	Despreciable
Válvula de globo totalmente abierta	60 - 300
Válvula de compuerta totalmente abierta	7
Válvula de compuerta abierta 3/4	40
Válvula de compuerta abierta 1/2	200
Válvula de compuerta abierta 1/4	800

Tabla.- Valores típicos de rugosidad relativa (e).

Material	**e (m)**
Concreto con acabado	5×10^{-5}
Concreto sin acabado	1.3×10^{-4}
Hierro colado	$1.5 - 2 \times 10^{-4}$
Ladrillo	$2 - 3 \times 10^{-4}$
Metal corrugado	$1.5 - 3 \times 10^{-3}$

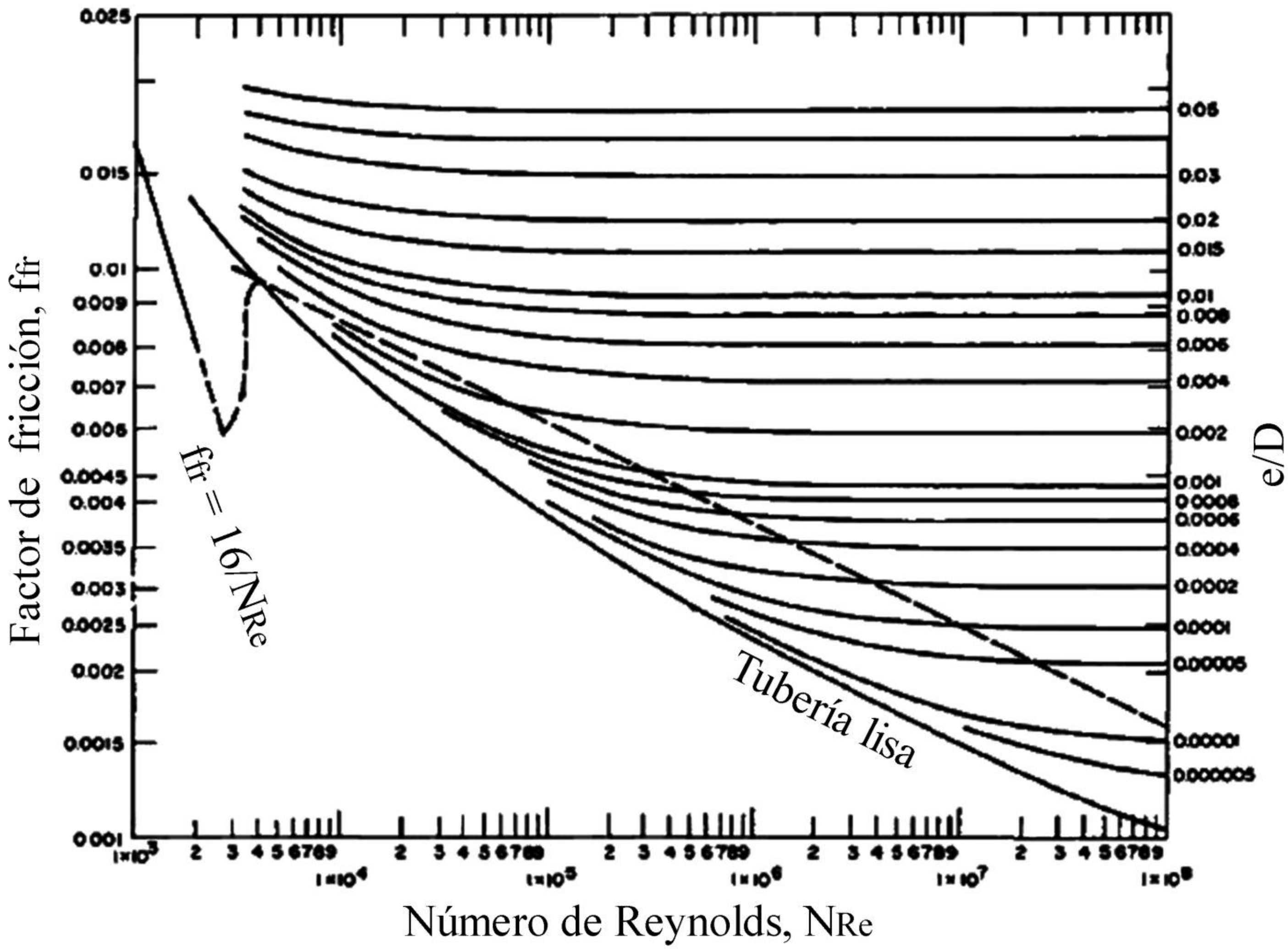

Figura.- Factor de fricción (f_{fr}) en función del número de Reynolds (N_{Re}) y la relación entre rugosidad relativa y diámetro de la tubería (e/D).